Contrastive Analysis of General and Special Forced Oscillations of Power Systems

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Abstract—With the continuous incorporation of renewable energy and new loads into the electric power grid, random factors that induce general forced oscillations (GFOs) gradually become risks that affect the power system’s security and stability. This research conducts a comparative analysis of the generation mechanisms of GFOs versus the traditional special forced oscillations (SFOs), specifically, from the perspective of frequency domain. Similarities and differences in en-oscillating conditions, occurrence probabilities, and the influencing factors of GFO and SFO are compared to better understand and recognize the GFO theory and the response characteristics of the power system under random excitations. A series of simulations in the 10-generator, 39-bus New England Test System is carried out to verify the analysis.

Index Terms—Forced oscillations, modal analysis, power spectral density (PSD), power system, random excitations.

I. INTRODUCTION

ARGE-SCALE power system interconnections that are gradually formed through regional power grid connections are the general trend worldwide [1]. Inter-regional power transmission can be realized through interconnection of regional power grids, and can bring about optimal allocation of a wide range of resources and emergency power support, thus improving the economic efficiency and reliability of power grid operation. However, with the expansion of the scale of the power grid, its structure and operation modes become increasingly complex, thus enhancing the difficulty of security and stability analysis, as well as power system controls. One common problem in all interconnected power grids is the oscillation phenomenon [2]–[4]. Its main performance characteristics are oscillations of constant amplitude or amplification. Oscillations not only limit the inter-area electric power transfer capability, but also seriously threaten the security and stability of power systems, inducing, at times, chain reactions and blackouts. Hence, conducting studies on oscillations are of great importance.

Increasing costs for primary energy sources in traditional electricity generation, such as coal and petroleum, accompanied by their adverse environmental impacts have led to extensive research on renewable energy sources in wind, offshore wind, and photovoltaic technologies. Similarly, when compared with other traditional sources, such as hydraulic power and thermal power, wind and photovoltaics present various advantages, particularly in terms of reproducibility of resources, as well as clean and environmentally friendly properties [5]. However, the obvious randomness of these sources inevitably becomes a hazard affecting the security and stability of power grids [6]–[8].

For oscillation generation in the actual power grid, researchers have provided several reasonable explanations based on theoretical analysis and oscillation reproduction mechanisms [9]–[12], such as weak damping, forced oscillation, and chaotic oscillation [13]–[16]. Corresponding measures to control oscillations of different mechanisms have also been adopted [17]–[21]. However, according to existing data, oscillations of unknown mechanisms still occur in interconnected power systems that are difficult to explain and control. Taking into consideration the range of uncertainties stemming from rapidly developing new energy generation sources, compounded by new loads such as electric vehicles, important questions arise such as—whether random excitations can induce new oscillation phenomena or which factors influence these new phenomena—all worthy of further discussion and research.

To research these issues, a mechanism analysis of power system oscillations caused by random excitations is investigated in frequency domains. The generation mechanisms between the proposed oscillations and the traditional forced oscillations are compared. Then, a comparative analysis of both is presented in this paper from different aspects such as en-oscillating conditions and influencing factors. The 10-generator, 39-bus New England Test System is used for calculation and analysis.

The remainder of the paper is organized as follows. Section II conducts a comparative analysis of the generation mechanisms of power system oscillations caused by random excitations versus the traditional forced oscillations. The en-oscillating conditions and occurrence probabilities of the two forced oscillations are compared in Section III. Section IV
describes similarities and differences in the influencing factors. Comprehensive case studies are also presented in this section. Finally, Section V provides some relevant conclusions.

II. COMPARISON OF OSCILLATION MECHANISM

In [15], the fundamental theory of power system forced oscillations is investigated in time domain. This work explains the mechanism of oscillations caused by a cyclic small excitation. However, forced oscillations caused by random excitations may have more complex and stochastic features, in which case the deterministic time-domain analysis method may be unsuitable. Hence, based on the power spectral density (PSD), the mechanism analysis of power system oscillations caused by random excitations in frequency domain is discussed in this section.

Let the frequency-domain transfer function of a linear system as $H(f)$, and PSD of the input stationary random process $u$ as $S_u(f)$, and PSD of the output stationary random process $y$ as $S_y(f)$. Based on the stochastic linear system theory, the linear system is modeled as (1) in [22]:

$$S_y(f) = |H(f)|^2 S_u(f).$$

Therefore, at a frequency or frequency band, only when $|H(f)|$ and $S_u(f)$ are large, $S_y(f)$ is large; when either $|H(f)|$ or $S_u(f)$ is 0 or very small, $S_y(f)$ is 0 or very small. In addition, previous studies have shown that $|H_{oi}(f)|^2$ can be depicted as a sharp wave when it is near the frequency $f_{oi}$ of one weak-damping oscillation mode $i$ [23]. If there are $I$ modes of this kind, the corresponding amplitude-frequency characteristics of $I$ modes present a relationship of product. In other words, the number of sharp waves that the amplitude-frequency characteristics has is equivalent to the number of weak damping modes. The solid lines in Fig. 1 are examples.

Assume that PSD of the input random excitation has remarkable amplitudes in some frequency bands, which cover the frequency of one weaker damping mode, as shown by chain dotted lines in Fig. 1(a). According to (1), PSD of the output variable after multiplying (as shown by dotted lines) becomes larger, such as a weaker damping mode $f_{o1}$ in Fig. 1(a); if PSD of the input random excitation is small, on the other hand, the PSD of the output variable at these frequency bands is small, such as weaker damping mode $f_{o2}$ and $f_{o3}$ in Fig. 1(a). Similarly, if the frequency band of the input random excitation changes, as shown in Fig. 1(b), PSD of the output variable is small at $f_{o1}$ and becomes large at $f_{o2}$ and $f_{o3}$. Hence, the generation mechanism of power system oscillations caused by random excitations can be obtained as follows: if frequency bands of the input random excitations can cover the inherent frequencies of some weaker damping modes in power systems, a large forced oscillation may be induced. The oscillation includes all frequency components of these modes.

Note that this mechanism can also be used to explain the traditional forced oscillation phenomenon in a particular way. If near the inherent frequencies of some weaker damping modes, a periodic excitation is input and its PSD is a vertical line, as shown by the chain dotted line in Fig. 2(a). According to (1), PSD of the output variable after multiplying is also a vertical line (as shown by the dotted line). In other words, the output variable is a periodic oscillation curve with the same frequency. Moreover, the closer to $f_{o3}$ the excitation frequency is, the larger the amplitude of the output PSD. On the other hand, if the excitation frequency shows obvious deviation from the inherent frequencies of weaker damping modes, the amplitude of the output PSD becomes small or even 0, as shown in Fig. 2(b).
where \( g(f) \) and \( \varepsilon(f) \) represent the amplitude-frequency characterist of the random excitation, which satisfy \( \frac{\varepsilon}{g} \ll 1 \).

If the frequencies of some weaker damping modes satisfy \( f_{oi} \in [f_m, f_M] \), an oscillation formed by all these frequencies is induced with high amplitude. This oscillation is defined as the general forced oscillation (GFO).

Particularly, if \( f_m = f_M \), the input is degraded into a continuing periodic excitation. Then, a large periodic oscillation can be excited under a rigorous condition of \( f_m = f_M = f_{oi} \). This oscillation is defined as the special forced oscillation (SFO).

Thus, different frequency characteristics of oscillations are determined, to a great extent, by the input excitation. If multimode frequencies are covered by PSD of random excitations, GFO is excited and contains all these frequencies. Thus, GFO can also be regarded as a multi-mode forced oscillation (MMFO). Correspondingly, if the input excitation only has one frequency, which is exactly equal to the inherent frequency of one weak damping mode, SFO or single-mode forced oscillation (SMFO) can be induced.

From comparing generation mechanisms of GFO and SFO, it can be concluded that GFO is an extension of SFO, and SFO is a particular case of GFO.

III. COMPARISON OF EN-OscILLATING CONDITIONS BETWEEN GFO AND SFO

The SFO theory indicates that continuing periodic perturbation can cause forced oscillations of power system [15]. When the disturbance frequency is equal to or near the natural oscillation frequency, or "point-to-point", the system resonance will be induced, resulting in forced oscillations with high amplitude. The forced oscillation has features like fast start-up, oscillation of constant amplitude after the start-up, and rapid attenuation after losing the disturbance source.

The GFO theory states that GFO may be induced when the frequency bands, observed in PSD of random excitations, can cover the inherent frequencies of some weaker damping modes of power system, and GFO is formed by all frequency components of these modes.

The widely used Gaussian white noise is a random process that its PSD is uniformly distributed in the entire frequency domain. This is an ideal random excitation. In the engineering field, random excitation is a random process with certain spectral width [24]–[26]. When the frequency band of the random excitation shares a “plane-to-point” correspondence with frequencies of some weaker damping modes, the oscillation of one or several modes may be excited.

Based on the comparative analysis of en-oscillating conditions between GFO and SFO, it can be demonstrated that the GFO theory significantly extends the en-oscillating conditions of SFO, and the occurrence probability is much larger than SFO.

IV. COMPARISON OF INFLUENCING FACTORS BETWEEN GFO AND SFO

A. Qualitative Analysis

For SFO, the input excitation and output response are single-frequency signals so that effects of different factors on the system output can be determined by intuitively comparing the
fluctuation range of time-domain curves. For a single-machine infinite-bus system, the oscillation amplitude of output variable can be deduced as follows [15]:

$$B = \frac{F_0}{K_S} \sqrt{(1 - \nu^2)^2 + (2\zeta\omega_0)^2} \quad (3)$$

where \(F_0\) is the amplitude of input; \(K_S = \left[E'U_\infty/X_\Sigma'\right]\cos\delta_0\) is the power coefficient; \(\zeta = D/2\omega_0T\) is the damping ratio; \(\nu\) is the ratio between the input excitation frequency \(f\) and natural oscillation frequency \(f_o\). Therefore, SFO is mainly influenced by the input excitation amplitude, system parameters, and operation state of the system.

For GFO, with the consideration of random excitations, the system response is a multi-frequency output signal, and the phase position of each frequency component is random. Therefore, the overlapped time-domain curve cannot be used to discriminate the influencing factors of GFO. To this end, the amplitude of the output PSD can be considered for the analysis. According to (1), the amplitude of the output PSD is influenced by the following two aspects, namely, the amplitude of the input PSD and the amplitude-frequency characteristics of weaker damping modes.

The amplitude of the input PSD is determined by the intensity of random excitations. In other words, the larger the disturbance intensity is put on the system, the greater the forced oscillation amplitude is excited. On the other hand, the amplitude-frequency characteristics corresponding to a certain oscillation mode reach the maximum value at frequencies of weaker damping modes, which satisfy [23]:

$$|H_{oi}(f_{oi})|^2 = \frac{C(f_{oi})}{\sigma_i^2(\sigma_i^2 + 16\pi^2 f_{oi}^2)} \quad (4)$$

where \(C(f_{oi})\) represents the coefficients from input to output for \(i = 1, \ldots, n - 1\); \(\sigma_i\) and \(f_{oi}\) are the attenuation factors and natural oscillation frequencies of oscillation modes, respectively. As shown in (4), the lower the frequency \(f_{oi}\) and the smaller the \(\sigma_i\) of the oscillation mode is, the larger the forced oscillation amplitude will be. Hence, under the same intensity of random excitations, if several weaker damping modes are excited, the lower-frequency and weaker-damping component corresponds to a higher amplitude in the output PSD.

In addition, \(C(f_{oi})\) in (4) is determined by the transfer function between the input excitation and output response, which is influenced by several factors, such as input and output locations, operation state of power system, and electrical distance between input and output in particular. It should be noted that in previous studies, as the oscillation of one weaker damping mode is excited, the effects of mode frequency on output are often neglected. However, random excitations may excite an oscillation of several oscillation modes. Thus, as one of the influencing factors that affect the amplitude of GFO, the natural oscillation frequencies of weaker damping modes should be concerned. For example, under the same conditions, the inter-area oscillation mode at a low frequency can cause greater harm to the power system than the local oscillation mode at a high frequency.

In order to analyze the influence of each factor, the quantitative calculation is conducted and presented as follows.

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**TABLE II**

<table>
<thead>
<tr>
<th>No.</th>
<th>Major Participant Generators</th>
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<tbody>
<tr>
<td>1</td>
<td>G4&lt;-&gt;G5</td>
</tr>
<tr>
<td>2</td>
<td>G7&lt;-&gt;G6</td>
</tr>
<tr>
<td>3</td>
<td>G8&lt;-&gt;G1</td>
</tr>
<tr>
<td>4</td>
<td>G1,G8&lt;-&gt;G9,G3,G6,G7</td>
</tr>
<tr>
<td>5</td>
<td>G2&lt;-&gt;G3</td>
</tr>
<tr>
<td>6</td>
<td>G5,G4&lt;-&gt;G5,G6,G7</td>
</tr>
<tr>
<td>7</td>
<td>G2,G3&lt;-&gt;G5,G6,G4,G7</td>
</tr>
<tr>
<td>8</td>
<td>G9&lt;-&gt;G5,G6,G3,G4,G7,G2</td>
</tr>
<tr>
<td>9</td>
<td>G10&lt;-&gt;G9,G6,G5,G4,G7,G3,G2,G1,G8</td>
</tr>
</tbody>
</table>

**B. Quantitative Analysis**

When the actual power system is a high-order nonlinear system, it is difficult to obtain an analytical expression of the frequency-domain transfer function \(\tilde{H}(f)\). Hence, a feasible way to analyze the influencing factors is by the time-domain simulation. Specifically, the simulation steps are described as follows.
Step 1 Set the frequency-band range and amplitude value of the input PSD; then use the Frequency-Time Transform Method to obtain time-domain sequences of random excitations [27], [28].

Step 2 Select the disturbance location and inject transformed sequences by using the current injection method [23]. Then, after choosing an observation variable, the oscillating curve can be calculated by transient stability analysis modules in PSASP.

Step 3 Make power spectrum analysis of the output time-domain curve, and for comparison, the peak amplitude of the output PSD and the corresponding frequency must be recorded.

The 10-generator, 39-bus New England Test System is shown in Fig. 3. Note that $G_{10}$ is an external equivalent generator, with a comparatively large inertia time constant. Therefore, $G_{10}$ is selected as the reference generator.

In the simulation system, the 5-order detailed model is adopted for generators, while the constant impedance model is used for loads. With the small interfering analysis modules in PSASP, 9 electromechanical oscillation modes and major participated generators can be obtained, as shown in Table I and Table II. The involvement degree of generators in each oscillation mode decreases progressively from left to right of both ends of the symbol (<->).

First, the effects of the oscillation mode frequency on the amplitude of GFO are analyzed. It can be seen from Table I, the damping ratios of mode 1 and 9 are similar, but the natural oscillation frequencies present remarkable differences. The amplitude-frequency characteristics of both modes are shown in Fig. 4. It can be seen that with close damping ratios of two oscillation modes, the amplitude-frequency characteristics between them at different frequencies present distinct differences. According to (1), the amplitude-frequency characteristics are directly correlated with the amplitude of the output PSD. Hence, the influence of mode frequency on GFO should be noted.

From Table II, $G_5$ is simultaneously involved in mode 1 and mode 9. Thus, it is considered that the bus 34 of $G_5$ should be selected as the equivalent current injection point to simulate the random fluctuations of active power. To stimulate the forced oscillation of both oscillation modes and exclude the influence of other modes, it can be assumed that the frequency-band range of random excitation is between 0.5–0.6 Hz (Coverage mode 9 frequency) and 1.4–1.5 Hz (Coverage mode 1 frequency). Then, the corresponding time-domain sequences are obtained as shown in Fig. 5.

The transformed time-domain sequences are injected in the simulation system, and the relative power angle of $G_5$ against $G_{10}$ is chosen as the output variable. After obtaining the time-domain curve of relative power angle, the power spectrum analysis is conducted in Fig. 6. According to Fig. 6, as the frequency bands of the input excitation cover the natural oscillation frequencies of modes 1 and 9, an oscillation concludes two mode frequency is excited. In addition, the oscillation amplitude at the frequency of mode 9 is greater than that of mode 1. By considering the analysis is based on Fig. 4, it can be summarized as follows: under the same intensity of random excitations, if the oscillation at frequencies of a number of weaker damping modes is excited, the amplitude of oscillation...
components at a lower frequency is higher, causing greater harm to the system. It should be noted that the input and output locations are the same in the above analysis, or the influence of electrical distance between input and output on oscillation amplitude is omitted and only the influence of oscillation mode is analyzed. In fact, electrical distance is another important factor that influences the response oscillation amplitude. This factor is then analyzed as follows.

According to Fig. 3, load bus 29, 28, 26, and 27 share a close electrical distance with G₉. Therefore, it is considered that these four buses can be selected as equivalent current injection points to simulate the random fluctuations of loads at different locations. Then, to make the electrical distance between input and output more intuitive and easy to compare, output observation points are replaced in G₉, which can also avoid influence on other generators. The relative power angle of G₉ against G₁₀ is selected as the output variable. Finally, assume the frequency band of the random excitation falls in 0.8–1.0 Hz to excite the mode 8 for the highly involved degree of G₉. PSD of the random excitation is shown in Fig. 7.

In Fig. 8, PSD of the relative power angle curve at different disturbance locations, i.e., bus 29 and 28, are shown. Under the same random excitation but different electrical distances between the excitation source and observation point, signifi-
cent difference of amplitudes is observed. With the random excitation unchanged, equivalent current is injected in the rest 2 load buses. Simulation results of the peak amplitude of the output PSD are recorded for comparison, as shown in Table III.

<table>
<thead>
<tr>
<th>Excitation Source Position</th>
<th>Amplitude of PSD</th>
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<tbody>
<tr>
<td>Bus 29</td>
<td>4.953</td>
</tr>
<tr>
<td>Bus 28</td>
<td>1.768</td>
</tr>
<tr>
<td>Bus 26</td>
<td>1.752</td>
</tr>
<tr>
<td>Bus 27</td>
<td>1.675</td>
</tr>
</tbody>
</table>

According to Table III, with the same random excitation, as the electrical distance between excitation source location and $G_0$ expanded, the amplitude of the output PSD continues to decrease. It can be concluded that the electrical distance between input excitation and output response can significantly affect the oscillation amplitude. The smaller the electrical distance, the more drastic the induced oscillation.

V. CONCLUSION

A comparison between GFO and SFO is summarized in this work, and listed in Table IV. The comparison shows the following:

1) GFO is a breakthrough and extension of SFO, and SFO is a specific case of GFO.
2) Compared with the “point-to-point” en-oscillating condition of SFO, “plane-to-point” en-oscillating condition of GFO significantly increases the possibility of system oscillation. This may explain the increased power system oscillation in recent time.
3) Among the influencing factors of forced oscillation, the electrical distance between input and output is of remarkable importance. The smaller the electrical distance, the more drastic the oscillation.
4) Among the influencing factors of GFO, the effect of oscillation mode frequency needs to be considered: the lower the frequency, the larger the oscillation component amplitude.

REFERENCES


<table>
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