# Volt/Var Optimizaiton for Active Power Distribution Systems on a Graph Computing Platform: An Paralleled PSO Approach

Jun Tan\*, Ming He<sup>§</sup>, Guofang Zhang<sup>§</sup>, Guangyi Liu\*, Renchang Dai\*, and Zhiwei Wang\* \*Global Energy Interconnection Research Institute North America, San Jose, CA 95134, USA §Sichuan Electric Power Corporation, Chengdu, China guangyi liu@geirina net

guangyi.liu@geirina.net

Abstract—This paper proposes a paralleled particle swarm optimization (PSO) algorithm based on MapReduce and graph computing mechanisms for real time Volt/Var optimization (VVO) application on large power distribution systems. As a nonlinear mixed integer programming problem, VVO is usually solved by artificial intelligent algorithms such as PSO. However, the PSO algorithm will suffer large computational burden when dealing with large systems. To achieve efficient solution for the VVO problem, the authors develop a paralleled PSO algorithm which can improve the computing speed dramatically. Moreover, this paper also proposes graph data model (GDM) for power distribution systems and apply graph computing to achieve fast power flow analysis for VVO application. The effectiveness of the proposed approach for VVO application has been validated on the IEEE 123 node test feeder considering unbalanced loading and line configurations. The computational efficiency of the proposed approach is also validated on various sizes of distribution systems.

# *Index Terms*—Volt/var optimization, paralleled PSO algorithm, active power distribution system, graph computing.

#### I. INTRODUCTION

VVO has drawn much attention recently as it is able to improve energy efficiency and improve power quality by reducing the losses and improving the voltage profile. VVO in active power distribution systems involves the control of multiple types of devices including: on-load tap changing (OLTC), voltage regulator (VR), capacitor bank (CB), and inverter-interfaced DGs.

As the VVO is essentially a complex nonlinear mixed integer programming problem, it is hard to solve by analytic methods. Nowadays, a lot of research has been conducted by using artificial intelligent algorithms such as ant colony optimization (ACO) [1], [2], genetic algorithm (GA) [3], [4], artificial neural networks [5], and PSO [6], [7], etc. Among them, PSO has achieved very promising results. However, artificial intelligence algorithms such as PSO will suffer large computational burden when dealing with large systems. For large power distribution system, high dimension nonlinear constraints of the VVO problem will result in a high dimension search space which will make the conventional PSO algorithm suffer the curse of dimensionality.

To achieve efficient optimization algorithm for VVO, this paper adopts graph data model (GDM) for power distribution systems and apply graph computing for both power flow analysis and power system optimization. GDM models power distribution system as a computing graph and store data in graph database. Thus, it is able to provide fast parallel power flow platforms, efficient data management approaches, and paralleled optimization algorithms. Graph computing is a game changing technology which constructs the power network from the viewpoint of a graph. The graph database also provides more efficient data management approaches by storing the information directly on vertices and edges. Many research has been conducted in the field of graph computing and its application in power systems [8]-[12]. However, very few study has applied the application of graph computing in power distribution system analysis and optimization. This paper models the power distribution network as a computing graph. Based on the GDM, a paralleled PSO algorithm has been proposed. The paralleled PSO algorithm is able to dramatically improve the computing speed, thus can provide an effective solution to the real time VVO application for large size distribution systems.

This study has made contributions in several major aspects by: (1) proposing a paralleled PSO approach for VVO application to improve the computational efficiency; (2) applying graph data model and graph computing in power flow analysis for power distribution systems; (3) applying graph computing for power distribution system optimization.

# II. PROBLEM FORMULATION

## A. VVO in Active Power Distribution Systems

In an active power distribution system the VVO includes the control of the OLTCs, VRs, CBs and DGs. The main objective of the VVO is to control the voltage in a normal range and reduce the system losses. Thus, the objective function can be formulated as:

This work was supported by State Grid Corporation technology project 5455HJ180018.

$$\operatorname{Min} \operatorname{O} \operatorname{F} = \alpha \cdot P_{loss} + (1 - \alpha) \cdot \sum_{i=1}^{n} |V_{base} - V_i|^2 \quad (1)$$

 $P_{loss} = \sum_{i=1}^{n} (P_{Gi} - P_{Li}) = \sum_{i=1}^{n} \sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} + \delta_{ij}) (2)$ s.b. to

$$V_{min} \le V_i \le V_{max} \tag{3}$$

$$P_{DG,i}^{min} \le P_{DG,i} \le P_{DG,i}^{max} \tag{4}$$

$$Q_{DG,i}^{\min} \le Q_{DG,i} \le Q_{DG,i}^{\max} \tag{5}$$

$$tap_i \in \{-tap_i^{min}, \dots, -1, 0, 1, \dots, tap_i^{max}\}$$
(6)

$$Q_{C,i} = \beta_C^i \cdot \Delta q_C^i \tag{7}$$

$$\beta_{C}^{i} \in \{0, 1, 2, \dots, \beta_{C, max}^{i}\}$$
(8)

where  $P_{loss}$  is the active power loss,  $V_{base}$  is the base voltage,  $V_i$  is the voltage at node *i*,  $P_{Gi}$  and  $P_{Li}$  are the active power generation and load at node *i*,  $P_{DG,i}$  and  $Q_{DG,i}$  are the active and reactive power generation of distribution generation at node *i*, tap<sub>i</sub> indicates the tap position of the LOTC or VR at node *i*,  $Q_{C,i}$  represent the reactive power output of the CB at node *i*, and  $\alpha$  is a weight coefficient.

#### B. Power Flow Formulation on Graph Computing Platform

The formulation VVO problem requires the power flow analysis to provide voltage profile and active power losses for obtaining the objective function regarding a potential control strategy. Thus, it is crucial to improve the power flow analysis efficiency to improve the efficiency of the optimization algorithm for real time application of VVO. In this Section, we will formulate a graph data model (GDB) for the power distribution system to apply graph computing for power flow analysis. Thus, we can use parallel computing approach based on graph computing and GDB to improve the power flow analysis efficiency.

The power distribution network can be formulated as a graph by mapping the components in the power distribution network to vertexes and edges in a graph. Accordingly, we can define the network as G(V, E) where V is the set of vertices and E denotes the set of edges for shown in Fig. 1. The load points, voltages regulators, switches and shunt capacitors can be modeled as vertices while the line segments and transformers can be modeled as edges. Both the vertices and edges have a set of attributes denoted as  $P(a_v, a_e)$ . Thus, the data for power flow computing and the computed results can be stored in the graph database.



Figure 1. Converting the distribution network into a computing graph.

The parallel mechanism of graph computing is shown in Fig. 2. As shown in the figure, there are three phases in the graph process engine (GPE). Phase 1 is for graph partition and resource allocation. Then, the phase 2 adopts the hierarchical group synchronization (HGS) parallel computing mechanism in bulk synchronous parallel (BSP) [13] and achieve node based parallel computing. Phase 3 is for synchronization and result output.



Figure 2. The parallel mechanism of graph computing.

As we will consider the three phase unbalanced power flow in power distribution system, the load and line configuration are unbalanced. We adopt a polynomial load model by considering the ZIP load as shown below:

$$\frac{P}{P_{0}} = a_{0} + a_{1} \left(\frac{|V|}{|V_{0}|}\right) + a_{2} \left(\frac{|V|}{|V_{0}|}\right)^{2}$$
(9)

$$\frac{Q}{Q_0} = b_0 + b_1 \left(\frac{|V|}{|V_0|}\right) + b_2 \left(\frac{|V|}{|V_0|}\right)^2$$
(10)

where  $a_0 + a_1 + a_2 = 1$ ,  $b_0 + b_1 + b_2 = 1$ 

Different load connection methods such as Wye and Delta connections are also considered as shown in (11) and (12).

$$\begin{bmatrix} S_{i}^{a} \\ S_{i}^{b} \\ S_{i}^{c} \end{bmatrix} = \begin{bmatrix} P_{i}^{a} + jQ_{i}^{a} \\ P_{i}^{b} + jQ_{i}^{b} \\ P_{i}^{c} + jQ_{i}^{c} \end{bmatrix} = \begin{bmatrix} (P_{o}^{a} + jQ_{o}^{a}) \left(\frac{|V_{i}^{a}|}{|V_{o}^{a}|}\right)^{n} \\ (P_{o}^{b} + jQ_{o}^{b}) \left(\frac{|V_{i}^{b}|}{|V_{o}^{b}|}\right)^{n} \\ (P_{o}^{c} + jQ_{o}^{c}) \left(\frac{|V_{i}^{c}|}{|V_{o}^{c}|}\right)^{n} \end{bmatrix}$$
(11)
$$\begin{bmatrix} S_{i}^{ab} \\ S_{i}^{bc} \\ S_{i}^{ca} \\ S_{i}^{ca} \end{bmatrix} = \begin{bmatrix} P_{i}^{ab} + jQ_{i}^{ab} \\ P_{i}^{bc} + jQ_{i}^{bc} \\ P_{i}^{ca} + jQ_{i}^{ca} \end{bmatrix} = \begin{bmatrix} (P_{o}^{ab} + jQ_{o}^{ab}) \left(\frac{|V_{i}^{ab}|}{|V_{o}^{bc}|}\right)^{n} \\ (P_{o}^{bc} + jQ_{o}^{bc}) \left(\frac{|V_{i}^{bc}|}{|V_{o}^{bc}|}\right)^{n} \\ (P_{o}^{ca} + jQ_{o}^{ca}) \left(\frac{|V_{i}^{ca}|}{|V_{o}^{ca}|}\right)^{n} \end{bmatrix}$$
(12)

The paralleled power flow method is to apply a MapReduce procedure on back-forward sweep algorithm. Specifically, Equation (9) and (10) update the voltages and currents in the forward sweep and equation (11) updates the voltages in the backward sweep [14].

$$[V_{abc}]_n = [a] \cdot [V_{abc}]_m + [b] \cdot [I_{abc}]_m$$
(9)

$$[I_{abc}]_n = [c] \cdot [V_{abc}]_m + [d] \cdot [I_{abc}]_m + \left(\frac{s_n}{[V_{abc}]_n}\right)^*$$
(10)

$$[V_{abc}]_m = [A] \cdot [V_{abc}]_n + [B] \cdot [I_{abc}]_m \tag{11}$$

where  $[V_{abc}]$  is the bus voltage matrix,  $[I_{abc}]$  is the line current matrix, [a], [b], [c], [d], [A] and [B] are the generalized 3 by 3 matrix which models the unbalanced series components and they are calculated as follows:

$$[a] = [U] + \frac{1}{2} \cdot [Z_{abc}] \cdot [Y_{abc}]$$
(12)

$$[b] = [Z_{abc}] \tag{13}$$

$$[c] = [Y_{abc}] + \frac{1}{4} \cdot [Y_{abc}] \cdot [Z_{abc}] \cdot [Y_{abc}]$$
(14)

$$[d] = [U] + \frac{1}{2} \cdot [Z_{abc}] \cdot [Y_{abc}]$$
(15)

$$[A] = [a]^{-1} \tag{16}$$

$$[B] = [a]^{-1} \cdot [b] \tag{17}$$

where [U] is the identity matrix,  $[Z_{abc}]$  is the impedance matrix, and  $[Y_{abc}]$  is the admittance matrix.

Fig. 3 shows the process of MapReduce in the three phase unbalanced power flow calculation of radial distribution network with backward forward sweep. In the backward forward sweep, the mapping process is to calculate the current node voltage and current, and the reduction process is to find out the father node voltage and current. Note that currents injecting to farther node set are calculated by aggregating branch currents which can be considered as Reduce phase. In forward sweep, node voltages are updated in a concurrent way as well while there is no current calculation involved in the sweep. We can surely predict that the computational performance will be better than sequential method.



Figure 3. The working principle of the graph computing based three-phase unbalanced power flow algorithm for power distribution networks.

## III. PARALLELED PSO APPROACH FOR VVO APPLICATION

# A. PSO Appraoch for Voltage/Var Optimization

As the objective function for the formulated VVO problem is a nonlinear mixed integer programming problem, analytical methods are not applicable. However, with artificial intelligent algorithm such as PSO, a near optimal solution can be found within a reasonable computing time. In this problem, the control variables including switchable shunts, controllable taps of voltage regulators and transformers, and output of DGs. PSO algorithm can easily map the continuous and integer variables into a search space. Specifically, the variable of each controllable device in the power distribution system can be naturally mapped into a dimension in the search space and the values of these variables can be viewed as coordinates in each dimension.

PSO algorithm [15] originates from the observation of the behaviors of fish schooling and bird flocking. In PSO algorithm, the possible solutions of a specific problem are mapped into a search space and the position of a particle in the search space is a potential solution to the problem. In the optimization process, particles are flying in the search space and their flying trajectories are determined by their current positions and velocities. The problem is to find the best positions of those particles which minimize the objective function during this process. The positions and velocities of the particles are updated each iteration according to (18)-(20). The objective function is used to evaluate the fitness of each potential solution. If a particle brings a better solution compared with its past visited positons, then the position of this particle is viewed as a personal best positon pBest. Similarly, if a better solution is reached compared with the past visited positions of all the particles, then the global best position gBest is found for current iteration. The algorithm will keep updating the gBest positions and values during the iterative process until an optimal solution is found.

$$v_{im}^{k+1} = wv_{im}^{k} + C_1 rand_1 \cdot \left(pBest_i - x_{im}^{k}\right) + C_2 \cdot rand_2 \cdot \left(qBest - x_{im}^{k}\right)$$
(18)

$$x_{im}^{k+1} = x_{im}^k + v_{im}^{k+1} \tag{19}$$

$$w = w_{max} - k \cdot \frac{w_{max} - w_{min}}{k_{max}}$$
(20)

where  $v_{im}$  represent the velocity of particle *i* at dimension m,  $x_{im}$  is the position of particle *i* in dimension *m*, *w* is the inertia weight, *k* is the iteration number, and  $C_1$ ,  $C_2$  are weight coefficients.

## B. Paralleled PSO Appraoch based on Graph Computing

Usually, artificial intelligence algorithms such as PSO will suffer large computational burden when dealing with large systems. For large power distribution system, high dimension nonlinear constraints of the VVO problem will result in a high dimension search space which will make the conventional PSO algorithm suffer the curse of dimensionality. To solve the problem, we will propose a paralleled PSO algorithm based on MapReduce mechanism and graph computing in this Section. Thus, the paralleled algorithm can leverage the available computing resources such as high performance servers or cloud computing to speed up the computing process.

MapReduce [16] uses a split-apply-combine strategy for assigning the work to different computing thread to achieve paralleled computing. The framework of the proposed paralleled PSO algorithm is shown in Fig. 4. As shown in the figure, MapReduce is applied to the PSO algorithm by dividing the algorithm into 'input', 'splitting/mapping', 'apply', 'reducing', and 'final result' five different phases. In the input phase, the positons and velocities for the particles are initialized. Then, each particle is mapped with a computing node which is handled by a worker in the computing resource partition process. The calculation of the computing nodes can be paralleled by the hierarchical group synchronization mechanism in bulk synchronous parallel. Thus, the computing process for each particle is ready for parallel computing. The third phase is the apply phase. In this phase, the PSO algorithm is applied for each particle to update its positon. In the reducing phase, each particle will update its *pBest* value. In the final result phase, the gBest value is updated and the algorithm is ready for next iteration or reaches a stopping criterial.



Figure 4. The framework of the proposed paralleled PSO algorithm

The computational procedure of the proposed paralleled PSO algorithm can be stated as follows:

Step 1: Initial the position  $x_{im}$  and velocity  $x_{im}$  for each particle in the search space.

Step 2: Map each particle to a computing node.

Step 3: Calculate the fitness value of each particle according to the objective function (1).

Step 4: Compare the fitness value with the current personal best value *pBest* of the particle and update *pBest*.

Step 5: Compare the fitness value with the current global best value *gBest* of all the particles and update *gBest*.

Step 6: Update the positions and velocities of all the particles according to (18)-(20)

Step 7: Check and limit the position  $x_{im}$  and velocity  $x_{im}$  within their maximum and minimum values.

Step 8: If the stopping criterion is reached, then go to Step 9; otherwise, go to Step 3.

Step 9: Output the optimal control strategy.

### IV. CASE STUDIES

# A. Simulation Environment

Case studies are carried out in this Section to validate the effectiveness of the proposed paralleled PSO algorithm for VVO application and further demonstrate its efficiency in computing time compared with conversional PSO algorithm. As shown in Fig. 5, the IEEE 123 node test feeder [17] is adopted in this simulation for VVO study. We modify the IEEE 123 node test feeder by adding 3 DGs at node 36, 51 and 72. The daily load profile is scaled from a residential load pattern in winter according to reference [18]. We use different size of the distribution system to test the computational speed of the proposed algorithm. The larger systems used in this study are obtained by combining multiple IEEE 123 node test feeders. The graph computing platform used in this study is TigerGraph v2.1 [19]. All the testing programs are implemented on a CentOS 6.8 server which has 2 CPUs × 6 Cores × 2 Threads @ 2.10 GHz with 64 GB memory.



Figure 5. IEEE 123 node test feeder

#### B. Simulation Results

The VVO is simulated in a 24 hour time horizon. The performance of the proposed paralleled PSO algorithm is compared with a base case without VVO as shown in Fig. 6. It can be observed from the results in Fig. 6 that the proposed approach is able to drive voltage near the rated voltage level to improve the voltage profile. The dynamics of the tap positions of the voltage regulators is shown in Fig. 7. The proposed approach is effective in VVO application as demonstrated through the simulation results.



Figure 6. Daily voltage profile of node 52 phase A of the test system



Figure 7. Tap position of the voltage regulators in the test system

Then, we can further investigate the computational efficiency of the proposed paralleled PSO approach based on MapReduce and graph computing. The computational time of the proposed paralledled PSO approach and the convensional PSO algorithm are tested with 4 different sizes of the test systems. The larger test systems are constructed with multiple 123 node test feeder by connecting their substation buses. The performance of the proposed paralleled PSO approach is shown in Fig. 8. It can be seen that the proposed paralleled PSO approach is able to dramatically reduce the computating time and the speed improvement is more significant with the increase of the system size. As a conclusion, the proposed method is able to overcome the drawbacks of the computational burden introduced by artifical intelligent algorithms and improve the computational speed significantly. It provides an effective solution for real time VVO application in large power distribution systems.



Figure 8. The computing time comparison of the proposed paralleled PSO approach and conversional PSO approach with different test systems

#### V. CONCLUSIONS

This paper builds the graph data model for power distribution system and apply graph computing for both the power flow analysis and distribution grid optimization. Specifically, the authors propose a paralleled PSO algorithm for volt/var optimization in an active power distribution system based on MapReduce and graph computing. The simulation results show that the proposed approach is able to overcome the computational burden of the conventional PSO algorithm and achieve fast optimization speed for real time VVO application for large power distribution systems.

#### REFERENCES

- F. S. Pereira, K. Vittori, and G.R.M da Costa, 'Ant colony based method for reconfiguration of power distribution system to reduce losses' *in Proc. Transmission and Distribution Conf. and Exposition*, Bogota, Colombia, August 2008, pp. 1–5.
- [2] A. Swarnkar, N. Gupta, and K. R. Niazi, 'Adapted ant colony optimization for efficient reconfiguration of balanced and unbalanced distribution systems for loss minimization', *Swarm Evolut. Comput.*, 2011, 1, (3), pp. 129–137.
- [3] E.R. Ramos and A.G Expósito, 'Path-Based distribution network modeling: application to reconfiguration for loss reduction,' *IEEE Trans. Power Syst.*, 2005, 20, (2), pp. 556–564.
- [4] J. Mendoza, R. López, and D. Morales, 'Minimal loss reconfiguration using genetic algorithms with restricted population and addressed operators: real application', *IEEE Trans. Power Syst.*, 2006, 21, (2), pp. 948–954.
- [5] H. Kim, Y. Ko, and K. H. Jung, 'Artificial neural-network based feeder reconfiguration for loss reduction in distribution systems', *IEEE Trans. Power Deliv.*, 1993, 8, (3), pp. 1356–1366.
- [6] M. Manbachi, H. Farhangi, A. Palizban, and S. Arzanpour, 'Smart grid adaptive energy conservation and optimization engine utilizing particle swarm optimization and fuzzification', *Applied Energy*, vol. 174, pp. 69-79, 2016.
- [7] H. Yoshida, K. Kawata, Y. Fukuyama, S. Takayama, and Y. Nakanishi, 'A particle swarm optimization for reactive power and voltage control considering voltage security assessment', *IEEE Trans. Power Sys.* Vol. 15, no. 4, pp. 1232-1239, Nov. 2000.
- [8] T. Werho, V. Vittal, S. Kolluri, and S. M. Wong, "Power system connectivity monitoring using a graph theory network flow algorithm", *IEEE Trans. Power Systems*, vol. 31, no. 6, pp. 4945-4952, Nov. 2016.
- [9] J. Jalving, S. Abhyankar, K. Kim, M. Hereld and V. M. Zavala, "A graph-based computational framework for simulation and optimisation of coupled infrastructure networks," in *IET Generation, Transmission & Distribution*, vol. 11, no. 12, pp. 3163-3176, Jul. 2017.
- [10] T. Ding, K. Sun, C. Huang, Z. Bie, and F. Li, "Mixed-integer linear programming-based splitting strategies for power system islanding operation considering network connectivity", *IEEE System Journal*, in press.
- [11] D. J. Won and S. II Moon, "Optimal number and locations of power quality monitors considering system topology", *IEEE Trans. Power Delivery*, vol. 23 no. 1, pp. 288-295, Jan. 2008.
- [12] P. Chavali and A. Nehorai, "Distributed power system state estimation using factor graphs", *IEEE Trans. Signal Processing*, vol. 63, no. 11, pp. 2864-2876, Jun. 2015.
- [13] Leslie G. Valiant, "A bridging model for parallel computation", *Communications of the ACM*, vol. 33, issue 8, Aug. 1990.
- [14] W. H. Kersting, Distribution system modeling and analysis, CRC Press, 2002.
- [15] J. Kennedy and R. C. Eberhart, "Particle Swarm Optimization," *IEEE International Conference on Neutral Networks*, Australia, 1995, pp. 1942-1948.
- [16] J. Dean and S. Ghemawat, "MapReduce: simplified data processing on large clusters", *Communication of the ACM*, vol. 51, no. 1, pp. 107-113. Jan. 2008.
- [17] "Distribution Test Feeders", 2017. [Online]. Available: https://ewh. ieee.org/ soc/pes/dsacom/testfeeders/.
- [18] J. A. Jardini, C. M. Tahan, and M. R. Gouvea, "Daily load profiles for residential, commercial and industrial low voltage consumers," *IEEE Trans. Power Delivery*, vol. 15, pp. 375-380, Jan. 2000.
- [19] "TigerGraph: The first native parallel graph", 2017. [Online]. Available: https://www.tigergraph.com/.