Security Region: An Intelligent Approach to Transportation Networks

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***Abstract*—This paper proposes a security region of transportation networks, which is defined as the set of flow states (operating points) satisfying the $N-1$ security in transportation networks. The boundary of the set is closed, inside all the operating points are $N-1$ secure; oppositely, outside all the operating points are insecure. First, $N-1$ security of the transportation networks is defined, which means that when a road is blocked, vehicles on other roads within the networks can also reach their destination through different paths without being trapped on the road. Then, the security region of transportation networks is modeled. The algorithm of a boundary calculation is studied and the boundary equations of security distances are proposed. Finally, the proposed security of the transportation networks is presented in different examples. This paper demonstrates the following: 1) the security region can be accurately predicted by the simulation of $N-1$; 2) the boundary distance can show the necessary degree of security; 3) security can be improved by a preventive control scheme. The simulation on TransCAD is used to verify the correctness of the results.**

***Index Terms*—Boundary distance, evolution, security region, transportation network, TransCAD.**

I. INTRODUCTION

TRANSPORTATION networks should be capable of responding to transportation operations, transportation environment and social public events [1]. The vulnerability of transportation networks is defined as the ability to maintain a stable operation when the transportation network service capacity sharply declines. The definition emphasizes the loss and effects on the random events and intentional attacks on network transportation [2]. The Events here include the harsh environment, the emergent public events, and daily transportation accidents [3]. Vulnerability is a new hot topic but also a difficult problem in the research of transportation networks. Bad service caused by a sudden attack can be avoided if the vulnerability can be reasonably considered in the planning, construction, operation and management of the transportation networks [4]. Studies on transportation network vulnerability are primarily focused on two aspects: one is to study how the unexpected events can influence network connectivity based only on network topology [5]; the other is to study the transportation efficiency variation when the transportation network is blocked [6].

This paper presents a novel method, called "security region", which is different from vulnerability. Security region is first applied in the research of the electric power grid [7]–[9]; the development of a smart grid makes the security region easier to use in researching other fields [10], [11]. This paper studies the application of security region in transportation network. The "security" here is not traffic safety. It means that the transportation network can guarantee a smooth operation instead of a large area of long time congestion caused by a single accident or contingency ($N-1$). If the flow state (operating point) can meet the security constraints above, it is said that the network is secure; otherwise it is insecure. In this paper, it is determined that there is a security boundary between the security operating points and the insecurity operating points of the transportation network. The boundary of the set is closed, inside all the operating points are $N-1$ secure; oppositely, outside all the operating points are insecure. A security region is determined to meet the maximum utilization of $N-1$ security constraints, which can be very convenient for security evaluation and optimization of the state of the network.

II. MODEL OF TRANSPORTATION NETWORK

According to graph theory [12], the main parameters of a transportation network are as follows:

1) Section and lane: a section is defined as a part of the road which is divided by crossroads. And a lane is defined as a part of the section for each vehicle at a moment in time. A section could contain 1–4 lanes.

2) Volume and saturation volume: as for a specific transportation network, each road is given two non-negative parameters: f_{ij} and c_{ij} (or labeled as f_A and c_A, where A represents a section of the road) represents the actual transportation flow and maximum transport transportation flow, which are called the flow of the section and saturated flow of the section. Saturated flow of the section is called capacity.

3) Degree of saturation: α_{ij}, the ratio of flow and capacity of a section. So it's clear that

$$\alpha_{ij} = f_{ij}/c_{ij} \tag{1}$$

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When the degree of saturation is too large, this section is too saturated.

In a topology network, the inflow of any node is equal to the outflow, which means the net flow of the node is zero.

$$\sum f_i = 0 \tag{2}$$

III. $N-1$ SECURITY OF TRANSPORTATION NETWORKS

A. Definition of $N-1$ Security

1) The operating point: the operational state of all the sections of the transportation network, including the speed of each section of the state, the flow, saturation, and etc.

The operating point of the network can be represented as a vector of 2N-dimension Euclidean space for the representation of the flow of each section. Among them, N is the total number of the section of the network; i and j mean nodes; f_{ij} means the flow of the section from i to j; v_{ij} means the speed of the section. The 2N-dimension vector is defined as (3):

$$P = (f_{12}, f_{13}, f_{23}, \cdots, f_{ij}, v_{12}, v_{13}, v_{23}, \cdots, v_{ij})^{\mathrm{T}} \tag{3}$$

2) $N-1$ security: the transportation network is operating at a point. When an accident is caused by any road blocking impassability ($N-1$ accident), after which the network can discharge distribution to meet the transportation demands, for the rest of all sections not experiencing flow saturation. If the operating point can meet the above constraints, it is said that the network at the operating point is $N-1$ secure; otherwise it is $N-1$ insecure. If the flow of the section is exactly equal to the capacity after the $N-1$ accident, the operating point is critical secure.

B. Transportation Redistribution after $N-1$ Accident

After a $N-1$ accident occurs, the vehicle on the network needs redistribution [13]. Different from the power system, transportation redistribution after a $N-1$ accident is not instantaneous, so time impedance must be considered. In this paper, based on the user equilibrium model of the transportation network [14], of which the core is that all the travelers in the transportation network choose the shortest path, mathematical models are determined as follows:

$$\min Z(f) = \sum_A \int_0^{f_A} t_A(w)\mathrm{d}w \tag{4}$$

$$\text{s.t.} \begin{cases} \sum_k f_k^{ij} = q_{ij}, & \forall i,j \\ f_k^{ij} \geq 0, & \forall i,j \end{cases} \tag{5}$$

In which

$$f_A = \sum_{i,j} \sum_k f_k^{ij} \delta_{A,k}^{ij},\ \forall A \tag{6}$$

and

$$t_A(f_A) = t_0[1 + \alpha(f_A/c_A)^{\beta}] \tag{7}$$

Equation (4) is the objective function, where $Z(f)$ is the total time impedance of all vehicles; f_A is the flow of Section A; t_A is the impedance of Section A; t_A (f_A) is the impedance function of Section A;

Equation (5) is the constraint condition, in which f_k^{ij} is the flow of path k between 2 points ($i\&j$); q_{ij} is the total flow between 2 points ($i\&j$).

In (6), $\delta_{A,k}^{ij}$ is a variable about the path: if section A is in the path k between the OD (i,j), the variable value is 1, otherwise it's 0;

Equation (7) is the impedance function, where t_0 is the impendence before the $N-1$ accident; α and β are constants: $\alpha = 0.15$, $\beta = 4$.

IV. DEFINITION AND ANALYSIS OF THE SECURITY REGION

A. Definition of the Security Region of the Transportation Network

The security region of the transportation network is defined as the set of all secure operating points while the topology and capacity of the transportation network are certain. The set has a unique closed boundary in a multidimensional space. All the operating points within the boundary are $N-1$ secure, and the operating points outside the boundary are $N-1$ insecure.

B. Calculation Method of the Security Boundary

In normal circumstances, the transportation flow of each section of the road should be less than the capacity. In any section of the $N-1$ accident, the flow of each section can be slightly overloaded after redistribution by applying (4) –(7), but there is still an upper limit. In this paper, 1.25 times the capacity is the limit. The section is considered saturated if the flow is above the limit. f_{mn} and c_{mn} stand for the flow and the capacity of each section in the shortest path. And now the expression of the security region under $N-1$ constraints is:

$$\Omega_{\mathrm{SR}} = \begin{cases} 0 \leq f_{ij} \leq c_{ij} \\ f_{ij} \leq \min(1.25c_{mn} - f_{mn}) \end{cases} \tag{8}$$

The boundary equations of the secure region are obtained by means of (8):

$$\begin{cases} B_1 : f_{ab} \leq \min(1.25c_{mn} - f_{mn}, c_{ab}) \\ B_2 : f_{bc} \leq \min(1.25c_{mn} - f_{mn}, c_{bc}) \\ \quad \cdots \\ B_n : f_{ij} \leq \min(1.25c_{mn} - f_{mn}, c_{ij}) \end{cases} \tag{9}$$

C. Formula of Security Boundary Distance

Security Boundary Distance is defined as the distance from the operating point to the security boundary. The calculation is:

$$D_{ij} = \min((1.25c_{mn} - f_{mn}) - f_{ij}, c_{ij} - f_{ij}) \tag{10}$$

Equation (10) represents that when there's an $N-1$ accident in any section, the minimum value of 1.25 times capacity minus the flow in each section is the security boundary distance after redistribution.

D. Security Evaluation Based on Region

If D_{ij} of all the sections are above 0, it can be determined that in the event of an $N-1$ accident, the network can still ensure normal operations, with no section saturated; if there is any D_{ij} below 0, indicating the occurrence of a $N-1$ accident, which can lead to the rest of the sections incurring supersaturating and congestion. The whole transportation network is not stable; if there are D_{ij} just equal to 0, the network is in a critical security state.

If the network is not secure, we can take certain measures to prevent and control. Adjust the distribution of transportation flow or limit part of the transportation flow, so that the operating point could return to the security region, and ensure that the network can be maintained after an $N-1$ accident.

V. EXAMPLE

A. Example Profile

The squares topology is used to verify the security region method. There are in total 12 sections, which are all with two-way four lanes. The widths of lanes are equal. In the same section, the capacities of positive and negative direction are equal. Its topology is shown in Fig. 1. The parameters are shown in Table I.

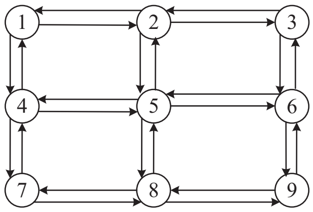

Fig. 1. Simple transportation network.

TABLE I
PARAMETERS OF THE SIMPLE NETWORK

Section	Length (km)	Bidirectional Capacity (pcu/h)
(1, 2)	20	2 417
(2, 3)	20	1 113
(1, 4)	10	3 877
(2, 5)	10	1 303
(3, 6)	10	1 258
(4, 5)	20	1 980
(5, 6)	20	1 551
(4, 7)	10	1 896
(5, 8)	10	733
(6, 9)	10	2 100
(7, 8)	20	1 896
(8, 9)	20	1 930

From Table I, it is can be seen that the length, capacity, and the shortest path of any section are available. For example, the length of the section (1, 2) is 20 km; the capacity is 2,417 pcu/h.

B. Security Assessment

1) Results of Security Evaluation

Security evaluation is for an operating point, which reflects a state of the section, including the transportation flow and the speed of the car.

Take the operating point P_{Y} for example. The parameters are in Table II.

TABLE II
SECURITY ANALYSIS DATA OF P_{Y}

Section	Speed (km/h)	α_{ij}	Volume (pcu/h)
(1, 2)	40	0.24	580
(1, 4)	37	0.22	852
(2, 1)	35	0.21	507
(2, 3)	35	0.31	345
(2, 5)	45	0.39	508
(3, 2)	38	0.24	267
(3, 6)	44	0.2	251
(4, 1)	40	0.17	659
(4, 5)	45	0.3	594
(4, 7)	52	0.19	360
(5, 2)	39	0.26	338
(5, 4)	38	0.24	475
(5, 6)	51	0.12	186
(5, 8)	50	0.34	249
(6, 3)	34	0.4	503
(6, 5)	43	0.22	341
(6, 9)	45	0.27	567
(7, 4)	34	0.23	436
(7, 8)	40	0.16	303
(8, 5)	41	0.21	153
(8, 7)	37	0.14	265
(8, 9)	43	0.18	347
(9, 6)	42	0.23	483
(9, 8)	45	0.24	463

At the operating point P_{Y}, when the section (1, 2) meets an $N-1$ accident, transportation flow will go along the shortest detour path 1-4-5-2 by (4)–(7). And in order not to cause saturation through the bypass path, by (8), (9), we can obtain:

$$\begin{cases} f_{12} \leq c_{12} = 2\,417 \\ f_{12} + f_{14} \leq 1.25c_{14} = 4\,846 \\ f_{12} + f_{45} \leq 1.25c_{45} = 2\,475 \\ f_{12} + f_{52} \leq 1.25c_{52} = 1\,628 \end{cases}$$

In summary, for the flow f_{12} of section (1, 2)

$$B_1 : f_{12} \leq \min(4\,846 - f_{14}, 2\,475 - f_{45}, 1\,628 - f_{52}, 2\,417)$$

In the same way, constraints after an $N-1$ accident in other sections of the road can be obtained, so as to obtain the security region expression of the whole transportation network:

$$B_1 : f_{12} \leq \min(4\,846 - f_{14}, 2\,475 - f_{45}, 1\,628 - f_{52}, 2\,417)$$
$$B_2 : f_{14} \leq \min(3\,021 - f_{12}, 1\,628 - f_{25}, 2\,475 - f_{54}, 3\,877)$$
$$B_3 : f_{21} \leq \min(1\,628 - f_{25}, 2\,475 - f_{54}, 4\,846 - f_{41}, 2\,417)$$
$$B_4 : f_{23} \leq \min(1\,628 - f_{25}, 1\,938 - f_{56}, 1\,391 - f_{63}, 1\,113)$$
$$B_5 : f_{25} \leq \min(3\,021 - f_{21}, 4\,846 - f_{14}, 2\,475 - f_{45}, 1\,303)$$
$$B_6 : f_{32} \leq \min(1\,391 - f_{36}, 1\,938 - f_{65}, 1\,628 - f_{52}, 1\,113)$$
$$B_7 : f_{36} \leq \min(1\,391 - f_{32}, 1\,628 - f_{25}, 1\,938 - f_{56}, 1\,113)$$
$$B_8 : f_{41} \leq \min(2\,475 - f_{45}, 1\,628 - f_{52}, 3\,021 - f_{21}, 3\,877)$$
$$B_9 : f_{45} \leq \min(4\,846 - f_{41}, 3\,021 - f_{12}, 1\,628 - f_{25}, 1\,980)$$
$$B_{10} : f_{47} \leq \min(2\,475 - f_{45}, 916 - f_{58}, 2\,370 - f_{87}, 1\,896)$$
$$B_{11} : f_{52} \leq \min(2\,475 - f_{54}, 4\,846 - f_{41}, 3\,021 - f_{12}, 1\,303)$$
$$B_{12} : f_{54} \leq \min(1\,628 - f_{52}, 3\,021 - f_{21}, 4\,846 - f_{14}, 1\,980)$$
$$B_{13} : f_{56} \leq \min(1\,391 - f_{23}, 1\,391 - f_{36}, 1\,628 - f_{52}, 1\,551)$$
$$B_{14} : f_{58} \leq \min(2\,475 - f_{54}, 2\,370 - f_{47}, 2\,370 - f_{78}, 733)$$
$$B_{15} : f_{63} \leq \min(1\,938 - f_{65}, 1\,628 - f_{52}, 1\,391 - f_{23}, 1\,113)$$

$$B_{16} : f_{65} \leq \min(1\,391 - f_{63}, 1\,391 - f_{32}, 1\,628 - f_{25}, 1\,551)$$
$$B_{17} : f_{69} \leq \min(1\,938 - f_{65}, 916 - f_{58}, 2\,412 - f_{89}, 2\,100)$$
$$B_{18} : f_{74} \leq \min(2\,370 - f_{78}, 916 - f_{85}, 2\,475 - f_{54}, 1\,896)$$
$$B_{19} : f_{78} \leq \min(2\,370 - f_{74}, 2\,475 - f_{45}, 916 - f_{58}, 1\,896)$$
$$B_{20} : f_{85} \leq \min(2\,370 - f_{87}, 2\,370 - f_{74}, 2\,475 - f_{45}, 733)$$
$$B_{21} : f_{87} \leq \min(916 - f_{85}, 2\,475 - f_{54}, 2\,370 - f_{47}, 1\,896)$$
$$B_{22} : f_{89} \leq \min(916 - f_{85}, 1\,938 - f_{56}, 2\,625 - f_{69}, 1\,930)$$
$$B_{23} : f_{96} \leq \min(2\,412 - f_{98}, 916 - f_{85}, 1\,938 - f_{56}, 2\,100)$$
$$B_{24} : f_{98} \leq \min(2\,625 - f_{96}, 1\,938 - f_{65}, 916 - f_{58}, 1\,930).$$

According to the security boundary results obtained before, by (10), the distance from operating point P_{Y} to the security boundary is calculated. The status data and security analysis results of P_{Y} are shown in Table III.

TABLE III
DISTANCE OF SECURITY BOUNDARY OF P_{Y}

Section	(1, 2)	(1, 4)	(2, 1)	(2, 3)
The shortest path after $N-1$	1-4-5-2	1-2-5-4	2-5-4-1	2-5-6-3
Security boundary distance (pcu/h)	502	189	433	512
Section	(2, 5)	(3, 2)	(3, 6)	(4, 1)
The shortest path after $N-1$	2-1-4-5	3-6-5-2	3-2-5-6	4-5-2-1
Security boundary distance (pcu/h)	380	723	614	446
Section	(4, 5)	(4, 7)	(5, 2)	(5, 4)
The shortest path after $N-1$	4-7-8-5	4-5-8-7	5-4-1-2	5-2-1-4
Security boundary distance (pcu/h)	119	217	516	135
Section	(5, 6)	(5, 8)	(6, 3)	(6, 5)
The shortest path after $N-1$	5-2-3-6	5-4-7-8	6-5-2-3	6-3-2-5
Security boundary distance (pcu/h)	340	1062	384	298
Section	(6, 9)	(7, 4)	(7, 8)	(8, 5)
The shortest path after $N-1$	6-5-8-9	7-8-5-4	7-4-5-8	8-7-4-5
Security boundary distance (pcu/h)	70	231	257	1021
Section	(8, 7)	(8, 9)	(9, 6)	(9, 8)
The shortest path after $N-1$	8-5-4-7	8-5-6-9	9-8-5-6	9-6-5-8
Security boundary distance (pcu/h)	352	294	198	144

Similarly, as for P_{N} and P_{C}, according to the security boundary results obtained previously, from (10), the distance from the two operating points to the security boundary are calculated. The state data and security analysis results of P_{N} and P_{C} are shown in Tables IV and V.

The security boundary distances of P_{Y} are all above 0, so this operation point is $N-1$ secure. 5 D_{ij} of P_{N} are negative, so P_{N} is not secure. As for P_{C}, there's $1 D_{ij}$ just equal to 0, whereas the other D_{ij} of the sections are above 0. So it is critical secure.

In order to observe the relative position of the operating point and the security region, use f_{32} and f_{36} for the independent variables, and the flow of the other sections for constants:

$$\begin{cases} 0 \leq f_{32} \leq 1\,113 \\ 0 \leq f_{36} \leq 1\,258 \\ f_{32} + f_{36} \leq 1\,391 \end{cases}$$

The relationship between the position of the three operating points and the security boundary is shown in Fig. 2.

In Fig. 2, the shadow part is the security region, P_{Y} is secure in the shadow; P_{C} is on the border, which is a critical secure operating point set; P_{N} is outside the shadow, and therefore insecure.

TABLE IV
SECURITY ANALYSIS DATA OF P_{N}

Section	Speed (km/h)	α_{ij}	Volume (pcu/h)	The shortest path after $N-1$	Security boundary distance (pcu/h)
(1, 2)	40	0.3	725	1-4-5-2	445
(1, 4)	37	0.21	814	1-2-5-4	87
(2, 1)	35	0.29	700	2-5-4-1	168
(2, 3)	35	0.6	667	2-5-6-3	132
(2, 5)	45	0.53	690	2-1-4-5	24
(3, 2)	38	0.6	667	3-6-5-2	115
(3, 6)	44	0.59	742	3-2-5-6	-13
(4, 1)	40	0.34	1318	4-5-2-1	26
(4, 5)	45	0.5	990	4-1-2-5	-114
(4, 7)	52	0.31	587	4-5-8-7	-27
(5, 2)	39	0.21	273	5-4-1-2	318
(5, 4)	38	0.21	415	5-8-7-4	95
(5, 6)	51	0.35	542	5-2-3-6	5
(5, 8)	50	0.5	366	5-4-7-8	728
(6, 3)	34	0.57	717	6-5-2-3	4
(6, 5)	43	0.32	496	6-3-2-5	161
(6, 9)	45	0.29	609	6-5-8-9	−42
(7, 4)	34	0.21	398	7-8-5-4	304
(7, 8)	40	0.35	663	7-4-5-8	−80
(8, 5)	41	0.12	87	8-7-4-5	958
(8, 7)	37	0.23	436	8-5-4-7	277
(8, 9)	43	0.27	521	8-5-6-9	217
(9, 6)	42	0.31	651	9-8-5-6	125
(9, 8)	45	0.23	443	9-6-5-8	75

TABLE V
SECURITY ANALYSIS DATA OF P_{C}

Section	Speed (km/h)	α_{ij}	Volume (pcu/h)	The shortest path after $N-1$	Security boundary distance (pcu/h)
(1, 2)	40	0.31	749	1-4-5-2	336
(1, 4)	37	0.27	1046	1-2-5-4	61
(2, 1)	35	0.42	1015	2-5-4-1	83
(2, 3)	35	0.3	333	2-5-6-3	565
(2, 5)	45	0.38	495	2-1-4-5	210
(3, 2)	38	0.55	612	3-6-5-2	128
(3, 6)	44	0.62	779	3-2-5-6	0
(4, 1)	40	0.17	659	4-5-2-1	400
(4, 5)	45	0.34	673	4-7-8-5	63
(4, 7)	52	0.23	436	4-5-8-7	127
(5, 2)	39	0.31	403	5-4-1-2	265
(5, 4)	38	0.24	475	5-2-1-4	99
(5, 6)	51	0.38	589	5-2-3-6	19
(5, 8)	50	0.41	300	5-6-9-8	741
(6, 3)	34	0.34	427	6-5-2-3	446
(6, 5)	43	0.22	341	6-3-2-5	298
(6, 9)	45	0.21	441	6-5-8-9	123
(7, 4)	34	0.35	663	7-8-5-4	70
(7, 8)	40	0.2	379	7-4-5-8	167
(8, 5)	41	0.21	153	8-7-4-5	1021
(8, 7)	37	0.14	265	8-5-4-7	352
(8, 9)	43	0.33	636	8-5-6-9	89
(9, 6)	42	0.23	483	9-8-5-6	198
(9, 8)	45	0.27	521	9-6-5-8	67

2) Security Evaluation Verification

The security analysis results of the three operating points are judged by the security region method. In order to verify the correctness of the results, the commonly used transportation simulation software TransCAD is used to carry out the $N-1$ simulation to the three operating points, one by one. The $N-1$ simulation results of each operating point are shown in Table VI.

In Table VI, "Y" means secure; "N" means insecure, "C" means critical secure.

The security simulation results in Table VI correspond to

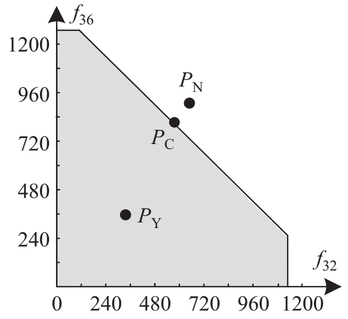

Fig. 2. Relationship between 3 points and security boundary.

TABLE VI
SIMULATION VALIDATION OF SECURITY REGION EVALUATION

Section	P_N	P_Y	P_C	Section	P_N	P_Y	P_C
(1, 2)	Y	Y	Y	(5, 6)	Y	Y	Y
(1, 4)	Y	Y	Y	(5, 8)	Y	Y	Y
(2, 1)	Y	Y	Y	(6, 3)	Y	Y	Y
(2, 3)	Y	Y	Y	(6, 5)	Y	Y	Y
(2, 5)	Y	Y	Y	(6, 9)	N	Y	Y
(3, 2)	Y	Y	Y	(7, 4)	Y	Y	Y
(3, 6)	N	Y	C	(7, 8)	N	Y	Y
(4, 1)	Y	Y	Y	(8, 5)	Y	Y	Y
(4, 5)	N	Y	Y	(8, 7)	Y	Y	Y
(4, 7)	N	Y	Y	(8, 9)	Y	Y	Y
(5, 2)	Y	Y	Y	(9, 6)	Y	Y	Y
(5, 4)	Y	Y	Y	(9, 8)	Y	Y	Y

the positive and negative of the security boundary distance in TABLES III–V. It can be seen that the results of the security evaluation based on the security region is persuasive.

C. Prevention and Control

Prevention and control are in order to ensure the stability of the sections before the transportation grooming [15]–[18] to ensure the stable operation of the transportation network. As a result, when an $N-1$accident in a section occurs, the entire network can maintain the normal state. Based on the theory of security region, the position of the operating points in the security region can provide the security information for the operating point, so that the dispatcher can understand the current transportation status and take preventive measures. The preventive control method is shown in Fig. 3.

Use the preventive control of P_N as an example. From Table IV, we can see thatsections (3, 6), (4, 5) and other sections are $N-1$ insecure. At this point, we need to take some preventive control measures to adjust the operating point to the security region.

Use section (3, 6) in P_N as an example. The transportation flow goes along the shortest path 3–2-5-6, and will lead to the section (2, 3) over saturation. From many simulations, we can determine to turn 52 pcu/hr from (3, 6) to the adjacent section (5, 6), so that the operating point returns to the security region. Figure 4. shows the variation of the operating points before and after the prevention and control.

Figure 5 is the comparison of the simulation results of $N-1$ before and after the prevention and control of section (3, 6).

It can be seen from Fig. 5(a) that when section (3, 6) is blocked after an $N-1$ accident, it cannot be accepted that the saturation of sections (2, 3) is above the upper limit of 1.25, which is over saturation; Fig. 5(b) is the result of after control measures, when the path (3, 6) is blocked, the vehicle goes through the shortest path, and will not cause any saturation in the sections.

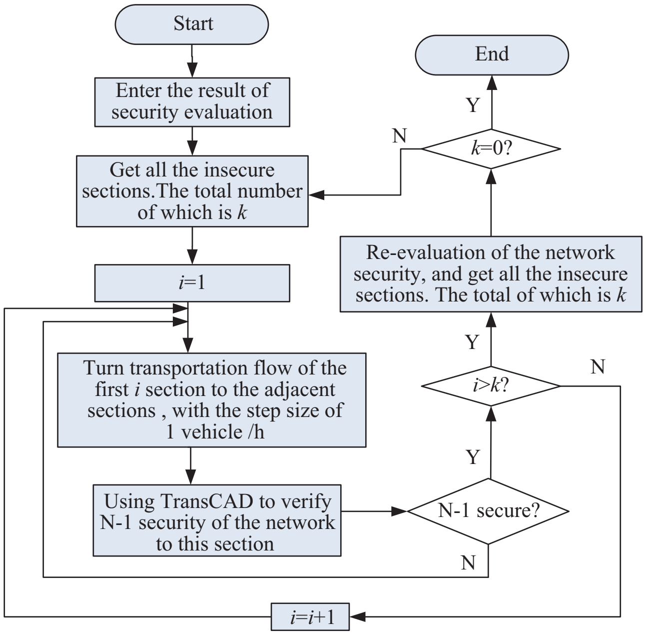

Fig. 3. Method of preventive control.

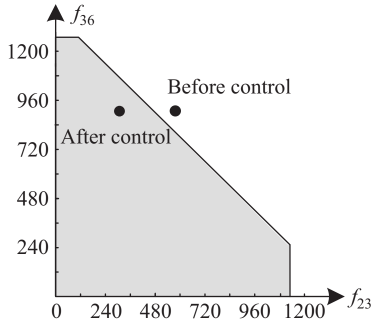

Fig. 4. The operating point returns to the security region.

Similarly, we can obtain the rest of the $N-1$ prevention and control programs, to take the security distance from the negative to positive, as is shown Table VII.

TABLE VII
RESULTS OF CONTROLLING THE BLOCKING ROAD OF P_N

Blocked road	(3, 6)	(4, 5)	(4, 7)	(6, 9)	(7, 8)
Security boundary distance before control/(km/h)	−13	−114	−27	−42	−80
Security boundary distance after control /(km/h)	21	16	13	57	32

After validation of TransCAD, the controlled network is secure for all sections after an $N-1$ accident, that is, an insecure operating point is turned into a secure operating point.

D. Instance Verification

Now the security region is applied to the transportation

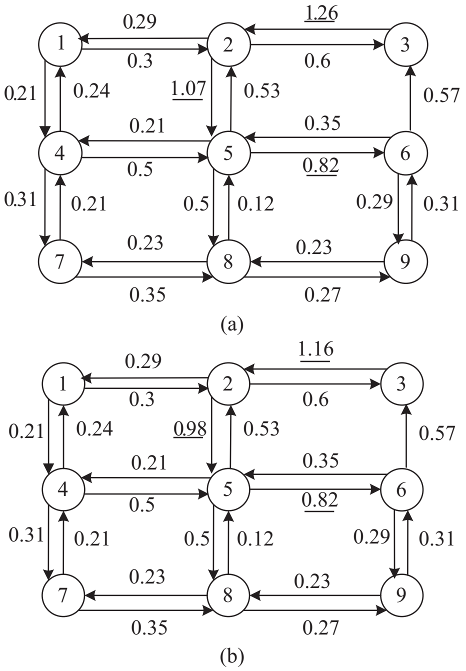

Fig. 5. Result of simulation changes after control. (a) Lead (2, 3) obstruction after the $N-1$ accident of (3, 6). (b) After prevention and control, other vehicle will bypass successfully after the $N-1$ accident of (3, 6).

network of Sioux Falls in USA [19]. The structure of the example is shown in Fig. 6. The parameters are shown in Table VIII.

Arrange three operating points P_{Y}, P_{N} and P_{C}. Due to space constraints, take P_{N} as an example for security region analysis, and the expression of the boundary of the security region is as follows:

$$\begin{cases} B_1: f_{12} \le \min(1\,218 - f_{13}, 891 - f_{34}, 926 - f_{45}, 257 \\ \quad -f_{56}, 258 - f_{62}, 1\,079) \\ B_2: f_{13} \le \min(1\,348 - f_{12}, 258 - f_{26}, 257 - f_{65}, 926 \\ \quad -f_{54}, 891 - f_{43}, 975) \\ \ldots \\ B_{17}: f_{8,16} \le \min(407 - f_{87}, 1\,218 - f_{7,18}, 272 \\ \quad -f_{18,16}, 210) \\ B_{18}: f_{16,18} \le \min(262 - f_{16,8}, 407 - f_{87}, 1\,218 \\ \quad -f_{7,18}, 218) \\ \ldots \\ B_{37}: f_{23,22} \le \min(272 - f_{22,21}, 254 - f_{21,24}, 264 \\ \quad -f_{24,23}, 208) \\ B_{38}: f_{23,24} \le \min(260 - f_{23,22}, 272 - f_{22,21}, 254 \\ \quad -f_{21,24}, 211) \end{cases}$$

Security analysis results of P_{N} are shown in Table IX.

There are 13 negative security distances in the table, which indicates that the operating point is $N-1$ insecure. After the control measures are taken, all the security boundary distances become positive.

The relationship between the local view and the operating point of the security region is shown in Fig. 7.

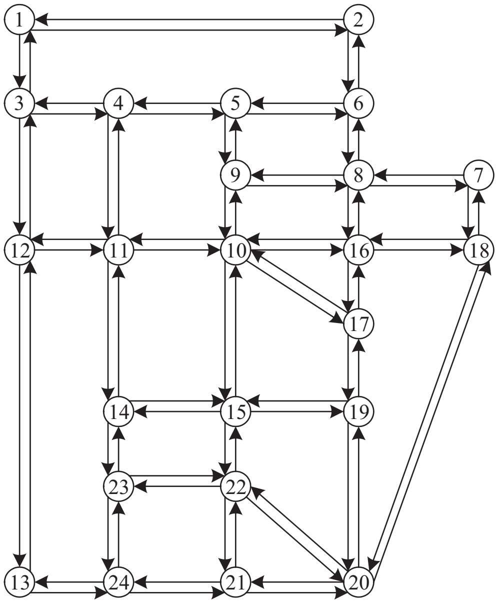

Fig. 6. Transportation network of Sioux Falls.

TABLE VIII
PARAMETERS OF SIOUX FALLS

Number	Section	Length (km)	Bidirectional capacity (pcu/h)
1	(1, 2)	6	1 079
2	(1, 3)	4	975
⋮	⋮	⋮	⋮
35	(8, 16)	5	210
36	(16, 18)	3	218
⋮	⋮	⋮	⋮
75	(23, 22)	4	208
76	(23, 24)	2	211

TABLE IX
SECURITY ANALYSIS RESULT OF P_{N}

Section	Speed	α	Volume (pcu/h)	The shortest path after $N-1$	Security boundary distance (pcu/h)
(1, 2)	32	0.12	140	1-3-4-5-6-2	17
(1, 3)	33	0.16	156	1-2-6-5-4-3	6
⋮	⋮	⋮	⋮	⋮	⋮
(8, 16)	35	0.64	136	8-7-18-16	4
(16, 18)	43	0.59	130	16-8-7-18	−3
⋮	⋮	⋮	⋮	⋮	⋮
(23, 22)	34	0.4	83	22-21-24-23	65
(23, 24)	45	0.35	73	23-22-21-24	65

VI. CONCLUSION

This paper proposes the concept and method of a security region for a transportation network, and provides a new method for analyzing the stability after an $N-1$ accident in a urban transportation network. The advantages of the method in this paper are as follows:

1) The security boundary can be calculated based on the topology of the transportation network and the speed of the vehicle, by which the stability of the networks can be precisely described and thus the operating point can be adjusted closer to the boundary in the area of security. This is useful to improve

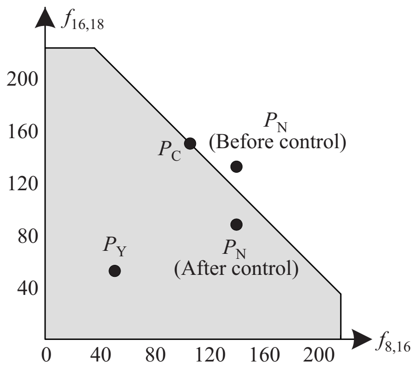

Fig. 7. Relationship between $f_{8,16}$ and $f_{16,18}$.

the potential capability and the utilization rate of transportation networks.

2) Security boundary distance can describe the security of a certain state of the network. The positivity and negativity of the distance shows the security of the network. The numerical value can quantify the security degree.

3) It is convenient for operators to determine preventive measures for the risk of large-area congestion before $N-1$ contingencies.

In this paper, the security region theory considers the topological structure, speed, capacity of the network, and transportation accidents under an external disturbance. In the future, we will consider the number and width of lanes on a section, and other factors in detail; and analyze the microscopic dynamic process of the vehicle, in order to obtain more accurate results for the security region.

With the development and wider use of intelligent transportation systems, the observability and controllability of transportation networks will be greatly improved. The concept of security region is suitable for this situation, and will provide a new method to learn situational awareness, advanced control and optimization of different types of networks.

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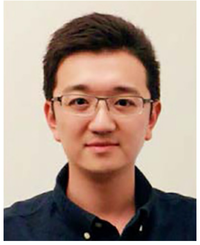

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