Loadability Assessment and Enhancement in Unbalanced Distribution Systems

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Abstract- Voltage stability is a major concern in heavily loaded distribution networks. Careful determination of control parameters for loadability enhancement may maximize the utilization of distribution networks. In literature most of the approaches optimize real/reactive power losses for the current operating condition of the distribution network. Although, these types of approaches increase the stability margin, such an increase may not be sufficient. The most important factor in loadability enhancement is representation of future load scenario in the optimization problem. Hence, in this paper a look ahead approach is developed for loadability enhancement of the unbalanced distribution system. The determination of critical loading point is conventionally done with continuation power flow, which is computationally very demanding, and also complex for implementation in unbalanced distribution networks. Hence, a new, computationally very efficient voltage stability index is developed here for determination of the loadability limit. The proposed methodology is demonstrated on IEEE 4 bus and 25 bus unbalance distribution systems with different transformer connections.

Index Terms- Maximum loading; Stability margin; Voltage stability index; Unbalance power flow.

I. INTRODUCTION

Due to increase in electricity demand during peak hours, power systems in many countries are forced to operate with low voltage stability margins. With the stressed operation of distribution networks voltage stability becomes a challenging constraint in both operational as well as planning aspects. The introduction of open competition among various utilities leads to congested networks at times, which further adds to the stability problems. In a strong power system voltage limits should be rigid. Voltage stability studies have always gained attention in research from time to time. A brief literature review for the same is presented below.


The power system attains the state of voltage instability due to an increase in load, a large disturbance, lack of reactive compensation or unsuitable location of compensating devices. The voltage stability issues can lead to serious blackouts in the system. The voltage stability can be enhanced by proper scheduling of control parameters as taps of transformer and shunts.

Toma et al. [6] proposed a voltage control methodology by tap changing transformer. In this work, changing the taps according to voltage variations on secondary side of the transformer controls the voltage. A tap is changed if the voltage moves away from a range between ±1.25%. Liu et al. [7] developed a nonlinear interior-point method for voltage control in distribution system by optimizing the tap setting of the transformer, number of switching operation of transformer LTC and capacitor.

Shunt capacitors are also used in voltage regulation of distribution systems. The capacitor banks can be represented as susceptances connected in either star or delta. The shunt capacitors can also be modelled as equivalent current injections [8, 9]. The current injection model of shunt capacitors can be comprised in backward/forward power flow process easily.

The determination of loadability limit is the first task while attempting steady state voltage stability analysis and enhancement. This is conventionally done with continuation power flow (CPF). Though CPF is proved to be most accurate, it is computationally very demanding. In case of unbalanced distribution network, it is very complex to implement. In some cases the power flow Jacobian is ill conditioned and the solution is not possible.

For quick and computationally cheaper calculation of voltage stability limit, many voltage stability indices have been proposed in the literature. But, most of the discussion is done for balanced distribution networks. Thus, it is seen from the literature that because of the special structure and features of the distribution system, blind extension of approaches suitable for transmission system, does not work well.

Hence, the first objective of this paper is to develop new, simple voltage stability indices for radial distribution networks. The developed indices are based on the percentage change in losses, and voltages, with increase in loading. These indices are independent of the type of power flow analysis used. Hence, a well-proven, three-phase forward backward power flow algorithm is utilized for its evaluation. Comparison of those results is done with conventional CPF to prove the accuracy of developed indices. Representation of future load scenarios is crucial to the success of any optimization technique. A look ahead approach is developed for voltage stability enhancement.
This type of approach facilitates the rescheduling of control variables for maximizing the loadability margin, looking at present and future operating conditions. Section II describes the framework for look ahead approach and details the three phase unbalanced power flow method. The mathematical modeling of voltage stability indices is explained in section III.

Section IV represents the mathematical model and algorithm for optimization. Taps of the transformers and shunts are taken as control variables. Improved harmony search algorithm is utilized for optimization. Section V presents the results and discussions. The work is concluded in Section VI.

II. FRAMEWORK FOR LOOK AHEAD APPROACH

In general, the real/reactive power optimization is carried at the present loading, and the control variables are rescheduled as per these requirements. In particular, reactive power controls are scheduled for minimizing the real or reactive power losses.

As mentioned earlier, many times power system is forced to operate near network loadability limit. In such a situation, close monitoring of the distance to collapse, and its enhancement become critical for power system operators. Also, the system behavior is quite nonlinear around the critical loading point. Hence, early bringing of reactive resources (or increase in reactive margin) may help in delaying this situation. For this, it becomes utmost important to represent load increase scenario in optimization process in some fashion and schedule the controls in an adaptive manner. Hence, in one step look ahead formulation, at each operating condition, the next load increase is predicted (forecasted). In this approach, at every load step the real power loss is optimized. It is then checked that with these controls whether meeting the next load step (from forecasting) is possible or not. If the load can be met at next load step, one resorts to the present load based optimization. However, if next load step is infeasible with the present setting of controls, then this optimization is not sufficient and the real power losses are optimized meeting the constraints at both the operating conditions. The complete framework can be depicted in Fig. 1.

A backward/forward power flow process that finds the current in each branch based on Kirchhoff current law and voltage at each bus based on Kirchhoff voltage law is reported in [10-13]. This paper utilizes a backward/forward process of unbalance distribution power flow [13]. The detailed process of power flow analysis is given in Appendix C.

The value of transformer admittance matrix is decided on the bases of three sub matrices \(Y_1, Y_2\) and \(Y_3\) [14]. Moreover, in backward and forward step the secondary current and primary voltage are modernized as in [15] to avoid the singularity problem of transformer solution.

III. NEW VOLTAGE STABILITY INDICES

Operators and analysts have observed that as the load increases, the bus voltages drop, and the system loss increases. Although, these changes are nonlinear, there is nothing abnormal about these when the loading is away from the critical loading. However, as we approach the critical loading, the voltage drop at the weakest bus, and the power loss in the most stressed line, increases abnormally. The proposed indices are designed to detect such a condition.

![Fig. 1. Look Ahead Approach.](image-url)
Where, $\lambda^{n-1}$ and $\lambda^n$ are the previous and current loading factor. $P_{L_{(p)}}^{n-1}$ and $P_{L_{(p)}}^{n}$ are the active power losses in a particular phase $p$ of the $l^{th}$ line corresponding to previous and current loading.

Loss based stability indicator is represented as

\[ VSI_{l=1p} = \frac{(PC_{l=1p})^2}{100} \]  

(4)

Where, $PC_{l=1p}$ the percentage change in active power losses in $l^{th}$ line (phase $p$). The major benefit of these indices is their availability almost without any additional computational cost, and also any power flow method can be utilized.

IV. OPTIMIZATION METHODOLOGY

A. Objective function

Four types of objective functions are considered in this study as follows

1) Minimizing active power loss

\[ Obj_{loss} = \min \left( \sum_{l=1}^{NL} \sum_{ph=a,b,c} P_{L_{(ph)}}^{ph} \right) \]  

(5)

Where, $P_{L_{(ph)}}^{ph}$ is the active power loss of $ph$ phase of $l^{th}$ line. $NL$ is the total number of line and $ph$ is the number of phases.

The set of equality and inequality constraints need to be satisfied are as follows:

Equality Constraint:
- The equality constraint is power balance i.e. total power injected at a bus should be equal to difference of total generation and demand at that bus.

\[ P_l^G - P_l^D = V_l \sum_{j=1}^{n} V_j (g_{ij}\cos\theta_{ij} + b_{ij}\sin\theta_{ij}) \]  

(6)

Where, $P_l^G$ is the total power generation and $P_l^D$ is the total power demand. $g_{ij}$ and $b_{ij}$ are real and imaginary part of $(i,j)^{th}$ entity of bus admittance matrix. As this study is carried out on distribution system, there is only one generation system which is substation.

Inequality constraints:
- The voltage should be within allowable limit. If $V_l^{ph}$ is the voltage of $l^{th}$ bus (phase $ph$), $V_l^{min}$ and $V_l^{max}$ are the minimum and maximum limits of voltages, then

\[ V_l^{min} \leq V_l^{ph} \leq V_l^{max}, \text{where} \; ph \in a, b, c \]  

(7)

- The control variable, shunts should be selected within permissible limit. If $Sh_j$ is the value of $j^{th}$ shunt, $Sh_j^{min}$ and $Sh_j^{max}$ are the minimum and maximum limits of shunts then

\[ Sh_j^{min} \leq Sh_j \leq Sh_j^{max} \]  

(8)

- Transformer taps setting should be within available range.

\[ Tap_k^{min} \leq Tap_k \leq Tap_k^{max} \]  

(9)

Where, $Tap_k$ is the tap setting of $k^{th}$ transformer and $Tap_k^{min}$ and $Tap_k^{max}$ are the minimum and maximum limits of transformer taps.

2) Minimizing reactive power loss

\[ Obj_{qloss} = \min \left( \sum_{l=1}^{NL} \sum_{ph=a,b,c} Q_{L_{(ph)}}^{ph} \right) \]  

(10)

Where, $Q_{L_{(ph)}}^{ph}$ is the reactive power loss of $ph$ line (phase $ph$). Subject to the constraints as described with previous objective, equation 6 to 9.

3) Maximizing the voltage stability indicator

Maximizing the voltage stability indicator, whenever value of next indicator is less than double of present indicator. This is achieved by, maximizing the difference of two successive voltage stability indicator (VSI) up to critical loading point, i.e. we are maximizing the critical loading point in an indirect way.

\[ Obj_{VSI} = \max \left( \left( VSI_l^{ph} \right)^{n+1} - \left( VSI_l^{ph} \right)^{n} \right) \]  

(11)

Subject to the constraints (6-9) and one additional constraint as:

\[ \left( \max(VSI_l^{ph}) \right)^{n+1} < 2 \times \left( \max(VSI_l^{ph}) \right)^{n} \]  

(12)

Where $NB$ is the total number of buses. $n$ and $n+1$ indicates the present and next loading states and ‘2’ reflects the unexpected change in VSI, that phenomena indicates the voltage collapse point, as observed in Section V (A).

4) Active power loss minimization with VSI as constraint

\[ Obj_{(loss, VSI)} = \min \left( \sum_{l=1}^{NL} \sum_{ph=a,b,c} P_{L_{(ph)}}^{min} \right) \]  

(13)

Minimizing the active power loss (13) with constraints in (6-9) and an additional constraint of voltage stability indicator as shown in (12). In this model the VSI is considered as a constraint in optimization processes to ensure the stability of the system for the available controls setting.

B. Look-ahead approach

In this process the objective functions are considered in two different ways for optimization [16]. First is the minimization of the considered objective at the present operating condition and second is near the critical loading point.

The look ahead optimization is carried out in this paper considering four different objective functions (5, 10, 11 and 13) to present a comprehensive analysis and applicability of the developed approach. The mathematical models for both the cases using objective function given by (5) are presented in the following sub sections. For other three objectives also, the equality and inequality constraints remains same as explained in Section IV (A).

i. Minimization of total active power loss at the present loading: This is the conventional present loading optimization. In this, the total active power losses in the lines are considered for optimization which can be represented as
\[
Obj = \min \left( \sum_{i=1}^{NL} \sum_{ph=a,b,c} P_{Loss_{ph}} (l) \right)
\]

Subjected to
Equation constraint
\[
P_l^g - P_l^p = V_i \sum_{j=1}^{n} V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij})
\]

Inequality constraints
\[
V_{i}^{\min} \leq V_{ia} \leq V_{i}^{\max}
\]
\[
V_{i}^{\min} \leq V_{ib} \leq V_{i}^{\max}
\]
\[
V_{i}^{\min} \leq V_{ic} \leq V_{i}^{\max}
\]
\[
U_{k}^{\min} \leq U_{k} \leq U_{k}^{\max}
\]

Where, \( Obj \) is the objective function to be minimized at present loading conditions. \( NL \) is the total number of lines in the system, \( ph \) is number of phases, \( P_{L}^{g} \) and \( P_{L}^{p} \) are power generation, demand and net injection at \( i^{\text{th}} \) bus, \( V_{i}^{\min} \) and \( V_{i}^{\max} \) are the minimum and maximum limit on \( i^{\text{th}} \) bus voltage, \( V_{ia} \) is the \( i^{\text{th}} \) bus (phase \( a \)) voltage at present loading, \( U_{k}^{\min} \) and \( U_{k}^{\max} \) are the minimum and maximum limits on \( k^{\text{th}} \) control variable. And (15) represents the present loading, power balance constraint.

ii. Minimization of total active power loss near critical loading: This is the new (look-ahead) optimization proposed especially for loadability enhancement. The objective is minimization of sum of series active power losses in the lines with voltage and control variable constraints at present (predicted) and previous operating conditions as
\[
Obj^{\acute{c}} = \min \left( \sum_{i=1}^{NL} \sum_{ph=a,b,c} P_{L_{Loss_{ph}}} (l) \right)
\]

Subjected to
\[
P_l^{p} - P_l^{g} = V_i \sum_{j=1}^{n} V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij})
\]
\[
V_{i}^{\min} \leq V_{ia} \leq V_{i}^{\max}
\]
\[
V_{i}^{\min} \leq V_{ib} \leq V_{i}^{\max}
\]
\[
V_{i}^{\min} \leq V_{ic} \leq V_{i}^{\max}
\]
\[
U_{k}^{\min} \leq U_{k} \leq U_{k}^{\max}
\]

Where, \( Obj^{\acute{c}} \) is the objective function near critical loading point. \( V_{ia} \) is the \( i^{\text{th}} \) bus (phase \( a \)) voltage at present loading, and \( V_{ia}^{\prime} \) is phase \( a \) \( \acute{c} \) voltage of \( i^{\text{th}} \) bus evaluated near critical loading point.

C. Improved Harmony Search Algorithm

In this problem controls are varying in discrete steps, so a meta-heuristic technique will be more feasible for optimization process. The control variables are optimized using improved harmony search algorithm (IHS). Z. W. Geem et al. [17] introduced a new meta-heuristic algorithm named Harmony Search (HS) that is a phenomenon conceptualized using mimicking musical process of searching for a perfect state of harmony. Pandi et al. [18, 19] proposed that if \( BW \) is the standard deviation of the population in each improvisation; it has better explorative power in improved harmony search algorithm. Hence, this version of IHS is utilized in present work. Fig. 2 describes the steps of improved harmony search algorithm. To check the worth of each solution, fitness function \( FF \) is evaluated as sum of objective function and penalties on constraint violations.

The IHS is utilized as optimization algorithm in this paper for demonstration however, the approach is general enough and any other algorithm can also be utilized.

![Fig. 2. Flow chart of improved harmony search algorithm.](image-url)
7. With these new controls, calculate new critical loading point ($\lambda_c^{n}$) using voltage stability index (2).
8. Check
   i. If $\lambda_c^{n} > \lambda_p$, Go to Step 2 and Optimize the controls at $\lambda = \lambda_p$.
   ii. If $\lambda_c^{n} < \lambda_p$, Be ready for emergency

V. RESULTS AND DISCUSSION

A. Voltage Stability Index

IEEE 4 Bus System

It consists of 4 buses and one transformer between bus 2 and 3. The base case load flow results are also validated with [20] and ETAP power flow solution.

Comparing the results for the above system for balanced and unbalanced test cases validate the proposed voltage stability indices with the ones given in [21]. In [21], Araujo et al. have developed a three phase continuation power flow. The results obtained for different type of transformer connections are shown in Table 1.

The maximum variation of voltage and loss based stability indicators with increase in system load for the balanced case is shown in Fig. 3 and 4 respectively. From Fig. 3 and 4 it can be observed that at critical loading point there is a steep change in voltage or loss profile that gives a clear indication of maximum loading point. That steepness could be at any of the bus or line according to its weakness. In the present case for the voltage based VSI maximum steepness is in phase a voltage of bus 4 and phase b of line 3 for power based stability indicator. Since both indices shows similar results, voltage based stability indicator is further utilized in the optimization process.

<table>
<thead>
<tr>
<th>Case</th>
<th>Type of Transformer connection</th>
<th>Type of Load</th>
<th>Maximum Loading Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Y_g - Y_g$</td>
<td>Balanced</td>
<td>33.1% (By [21]) 33% (Proposed)</td>
</tr>
<tr>
<td>2</td>
<td>$Y_g - Y_g$</td>
<td>Unbalanced</td>
<td>13.2% 13%</td>
</tr>
<tr>
<td>3</td>
<td>$\Delta - Y_g$</td>
<td>Balanced</td>
<td>34.8% 32%</td>
</tr>
<tr>
<td>4</td>
<td>$\Delta - Y_g$</td>
<td>Unbalanced</td>
<td>14.9% 15%</td>
</tr>
</tbody>
</table>

Fig. 5 and 6 shows the voltage profile of $Y_g - Y_g$ balanced and unbalanced cases at different buses. It can be observed from the figures that in $Y_g - Y_g$ balanced case, voltage of phase ‘a’ is decreasing, while phase ‘b’ and ‘c’ voltage shows increasing behavior up to maximum loading point. In $Y_g - Y_g$ unbalanced case, phase c has the lowest voltage because of the highest load connected to that phase. Similarly, the voltage profiles for $\Delta - Y_g$ balanced and unbalanced cases are shown in Fig. 7 and 8 respectively, which are exactly matching with the ones reported in [21].

Fig. 3. Voltage based VSI of IEEE 4 bus $Y_g$-$Y_g$ balanced case at bus 4 (phase A).
Fig. 4. Loss based VSI of IEEE 4 bus $Y_g$-$Y_g$ balanced case at line 1 (phase B).
Fig. 5. Voltage profile of IEEE 4 bus $Y_g$-$Y_g$ balanced case (a) Voltage at bus 2 (b) Voltage at bus 4

5 (a)
5 (b)
Fig. 6. Voltage profile of IEEE 4 bus Yg-Yg unbalanced case (a) Voltage at bus 2 (b) Voltage at bus 4.

Fig. 7. Voltage profile of IEEE 4 bus Δ-Yg balanced case at bus 2.

Fig. 8. Voltage profile of IEEE 4 bus Δ-Yg unbalanced case at bus 2.

1. Balanced distribution system

10 Bus System

A 10 bus balanced distribution system is considered for the study. It contains 9 load points and total system load is 12.368 MW. Single line diagram of the network is presented in Fig. 9. Three shunts of maximum capacity 0.08 p.u. are considered in the system at bus 8, 9 and 10. Table 2 shows the variation in controls for each case when objective functions (5, 10, 11, 13) are considered respectively.

It is clear from the Table 2 that all the objectives are giving almost similar results but the computational efforts are minimum with objective in (5). The performance time of each objective function is shown in Table 3. The objective in (5) has lower performance time, so the objective of (5) is used in further studies.

Table 3 Performance time of different objective functions

<table>
<thead>
<tr>
<th>Objective Function</th>
<th>(Obj_{Ploss}^{(5)})</th>
<th>(Obj_{Qloss}^{(10)})</th>
<th>(Obj_{VSI}^{(11)})</th>
<th>(Obj_{(Ploss, VSI)}^{(13)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance Time (sec.)</td>
<td>1207.89</td>
<td>1227.2</td>
<td>2469.36</td>
<td>2971.72</td>
</tr>
</tbody>
</table>

52 Bus System

An 11 KV, 52 bus practical distribution system [23] is also used in study. Fig. 10 shows the schematic diagram of 52 bus distribution system [23] and IEEE 4 bus and modified 25 bus [24, 25] unbalanced distribution systems.

Three cases in each system configuration are considered. a) an un-optimized case, this is subjected to the condition that controls are not available or set at initial position b) present loading optimization: in this the load is increased in uniform manner and controls are optimized c) look-ahead approach: in this, controls are optimized looking at one step ahead.

B. Look-ahead approach

The loadability enhancement with look ahead approach is attempted for standard 10 bus balanced [22], 52 bus practical...
distribution system. The shunts are considered at bus 44, 50 and 52. The locations of shunts are calculated by performing a voltage sensitivity analysis. As stated in earlier section; the objective of (5) is used in this study. Table 4 shows that loadability of the system is enhancement using look-ahead approach by 35.135% from un-optimized case and by 10.811% from present optimization case.

II. Unbalanced distribution systems with transformer

Due to the lesser computational efforts, active power loss minimization is utilized as objective for IEEE 4 bus and modified 25 bus unbalanced distribution systems. The detailed mathematical model is presented in section IV B in (14) and (20). For IEEE 4 bus and modified 25 bus unbalanced system, total nine combinations of transformer connections are possible as describe in [16]. In this paper, due to the limitation of space results are shown only for four combinations taking care of all type of transformer winding connections (Y, Yg, Δ).

The on-load-tap-changer (OLTC) transformers are used to maintain the voltage levels. In this work voltage at the secondary winding is controlled. The tap changers positions are taken in discrete steps. The voltage limits are assumed to be +/-10%. The shunt capacitors are considered as susceptance connected in star or delta at the nodes on the basis of transformer connections. The prediction of next load increase is taking as increasing load linearly with a step size 0.1 for 4 bus system and 0.2 for 25 bus system.

4 Bus System

Table 5 shows the variation of control parameters for all three cases with different transformer connections. For this system, the taps are tuned between 0.95 to 1.05 in discrete steps and two shunts of maximum 0.8 p.u. are considered at bus 3 and 4. The results shown in Table 5 depicts that the maximum loadability enhancement is achieved with look-ahead approach. The maximum loadability can further be increased by repeating steps 2 to 8 of look-ahead approach till all the controls are exhausted.

25 Bus System

Fig. 11 shows the single line diagram of modified 25 bus test system. The modification is shown in Appendix. One transformer is considered at substation side in standard 25 bus distribution system [24]. Hence, actual test system used in this work has 26 bus and 24 branches. The base case results for standard 25 bus system are validated with [24] and ETAP unbalanced load flow module.

In modified system, the shunts are considered at bus 13, 23 and 26. The locations of shunts are calculated by performing a voltage sensitivity analysis. Table 6 shows the variation of control parameters for un-optimized, present loading optimization and look-ahead approach with different transformer connections. It can be observed from Table 6 that maximum loading is achieved with look ahead approach.
the proposed approach. The obtained results demonstrate the potential of distribution networks. Results are obtained for three standard implemented for balanced and three phase unbalanced of the distribution networks. Look ahead approach is proposed for loadability enhancement catering more demand without loss of voltage stability. Hence, failure of power system. Increase in loadability limit helps in the proposed indicators.

Due to increased loading, voltage collapse may cause the failure of power system. Increase in loadability limit helps in catering more demand without loss of voltage stability. Hence, a look ahead approach is proposed for loadability enhancement of the distribution networks. Look ahead approach is implemented for balanced and three phase unbalanced distribution networks. Results are obtained for three standard test systems. The obtained results demonstrate the potential of the proposed approach.

APPENDIX

A. Modification in 25 bus data:

Transformer is incorporated between modified bus and bus 1 in standard 25 bus distribution system.

The leakage impedance of transformer is 0.025+0.24i Maximum and minimum limit of load bus voltages is ±10% i.e. 1.10 PU and 0.9 PU.

The shunts are considered at bus 13, 23, 26 with maximum availability of 0.2 PU.

The minimum and maximum limit of transformer tap is 0.95 PU and 1.05 PU.

B. Parameters for Improved Harmony Search Algorithm:

Harmony memory consideration= 0.98
Pitch adjustment= 0.1 to 0.99
HMS= 20
Maximum number of iterations= 1000

C. Distribution System Power Flow

I. Transformer Models and Handling

<table>
<thead>
<tr>
<th>Transformer Connection</th>
<th>Case</th>
<th>Controls</th>
<th>Maximum Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Tap</td>
<td>Shunt (13)</td>
</tr>
<tr>
<td>$Y_g − Y_g$</td>
<td>Un-optimized</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>Present optimization</td>
<td>0.9500</td>
<td>0.190</td>
</tr>
<tr>
<td></td>
<td>Look-ahead approach</td>
<td>0.9500</td>
<td>0.190</td>
</tr>
<tr>
<td>$Y − Δ$</td>
<td>Un-optimized</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>Present optimization</td>
<td>0.9500</td>
<td>0.150</td>
</tr>
<tr>
<td></td>
<td>Look-ahead approach</td>
<td>0.9626</td>
<td>0.170</td>
</tr>
<tr>
<td>$Δ − Y_g$</td>
<td>Un-optimized</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>Present optimization</td>
<td>0.9500</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>Look-ahead approach</td>
<td>0.9500</td>
<td>0.190</td>
</tr>
<tr>
<td>$Δ − Δ$</td>
<td>Un-optimized</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>Present optimization</td>
<td>0.9500</td>
<td>0.150</td>
</tr>
<tr>
<td></td>
<td>Look-ahead approach</td>
<td>0.9563</td>
<td>0.180</td>
</tr>
</tbody>
</table>

VI. CONCLUSIONS

Voltage collapse begins at the load ends under stressed conditions. Hence, assessing and enhancing voltage stability in distribution system is important. Since the straight forward extension of the transmission system methods may not work well for distribution systems, new and simple voltage stability indices have been developed in the paper. These are then used in different ways for voltage stability enhancement. Results for indices have been developed in the paper. These are then used well for distribution systems, new and simple voltage stability extension of the transmission system methods may not work distribution system is important. Since the straight forward

In general, a three phase two winding transformer is represented by two coupled coils as shown in Fig. 12.

![Fig. 12. Three phase two winding transformer.](image)

Where, \[ \begin{bmatrix} I_p \\ I_s \end{bmatrix} = \begin{bmatrix} Y_{pp} & Y_{ps} \\ Y_{sp} & Y_{ss} \end{bmatrix} \times \begin{bmatrix} V_p \\ V_s \end{bmatrix} \] (29)

The value of $Y_{pp}$, $Y_{ps}$, $Y_{sp}$ and $Y_{ss}$ can be decided based of three sub matrices $Y_1$, $Y_2$ and $Y_3$. Table 7 shows submatrices of three phase transformer step down connection [14].

<table>
<thead>
<tr>
<th>Winding Connection</th>
<th>Self-Admittance</th>
<th>Mutual Admittance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Secondary</td>
<td>Primary Secondary</td>
<td>Primary Secondary</td>
</tr>
<tr>
<td>$Y_{pp}$</td>
<td>$y_{pp}$</td>
<td>$Y_1$</td>
</tr>
<tr>
<td>$Y_{ps}$</td>
<td>$y_{ps}$</td>
<td>$Y_1$</td>
</tr>
<tr>
<td>$Y_{sp}$</td>
<td>$y_{sp}$</td>
<td>$Y_1$</td>
</tr>
<tr>
<td>$Y_{ss}$</td>
<td>$y_{ss}$</td>
<td>$Y_1$</td>
</tr>
</tbody>
</table>

Where, $Y_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $Y_2 = \frac{1}{3} \begin{bmatrix} 2y_t & -y_t & -y_t \\ -y_t & 2y_t & -y_t \\ -y_t & -y_t & 2y_t \end{bmatrix}$ and $Y_3 = \frac{1}{\sqrt{3}} \begin{bmatrix} -y_t & y_t & 0 \\ y_t & -y_t & 0 \\ 0 & 0 & -y_t \end{bmatrix}$ and $y_t$ is the transformer leakage admittance in per unit.
II. Singularity Problem Handling [15]

Some reforms are needed to accommodate the distribution transformer in backward/forward algorithm and avoid the singularity problem of transformer solution. In backward and forward step the secondary current and primary voltage can be computed as follows:

\[
\begin{align*}
(I_s - Y_{ss} V_s) &= Y_{sp} V_p \\
(I_p - Y_{pp} V_p) &= Y_{ps} V_s
\end{align*}
\] (30)

For all the connection of transformer except \( V_g - Y_g \) connection, equation can be replaced as:

\[
\begin{align*}
V_p &= V_p^{(1+2)} + V_p^{0} \\
V_p^{(1+2)} &= Y_p^{-1} (I_s - Y_{ss} V_s) \\
V_p^{0} &= \frac{(V_p^2 + Y_{pp} V_p + Y_{ps} V_s)}{3}
\end{align*}
\] (31)

Where, \( V_p^{(1+2)} \) contains the positive and negative sequence component and \( V_p^{0} \) is the zero sequence component of voltages. \( V_s \) and \( I_s \) are obtained by putting zero in their last row. \( V_p \) is obtained by replacing last row of \( Y_{sp} \) with 1. Similar steps are followed in both, backward and forward.

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