Comparing the Competition Equilibrium with the Nash Equilibrium in the Electric Power Market

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***Abstract*—The Nash equilibrium and competition equilibrium have been widely studied in the electric power market up to now. In this paper, it is explained that the Nash equilibrium can be achieved by using marginal cost pricing and the competition equilibrium can be performed by using accounting cost pricing based on the model of the power market system. The comparison between the Nash equilibrium and competition equilibrium indicates that surplus and unfair allocation of market benefits may be obtained by the Nash equilibrium, and the competition equilibrium realizes the optimization in economics with maximum market efficiency and fairness for market benefit allocations while the optimization in mathematics is achieved by the Nash equilibrium. There is sameness between the Nash equilibrium and competition equilibrium at the point when the power network characteristics are disregarded. The case study is made on an IEEE 30-bus system, and the calculation results indicate that it is the key issue to perform the competition equilibrium by using accounting cost pricing.**

***Index Terms*—Accounting cost pricing, competition equilibrium, marginal cost pricing, market failure, market surplus, Nash equilibrium, power market.**

I. Introduction

THE electric power market indicates that the operation and management of the power network and system should follow the economic principles so that the power resources can be allocated more efficiently. In other words, the “electric power market” is the combination of the rules in power systems and principles resulting from the economy.

The researches, most of which are trying to apply economic principles into every aspect of the electric power market, have been conducted all over the world. Equilibrium theory is one of the important foundation theories in the power market research, which can be classified into the Nash equilibrium [1] and Walrasian equilibrium [2].

The early market equilibrium proposed by Walras was called “Walrasian equilibrium” [3]. But the existence of general equilibrium was not convincing enough. Furthermore, Arrow and Debreu reconstructed the theory foundation of general economic equilibrium and provided a satisfying and strict mathematical proof of the general equilibrium existence in 1954 [4]. Meanwhile, Nash proved the existence of the Nash equilibrium in N-players game theory by using the fix-point theorem in 1950 [5]. The existence of the Nash equilibrium with additional constraints is the key point to obtain the Arrow-Debreu equilibrium. So we can conclude that the market equilibrium theory develops along this process: Walrasian equilibrium – Nash equilibrium – Arrow-Debreu equilibrium.

In the past years, Game theory and different Nash equilibrium models have been used in the analysis of strategic interaction among participants in the power market, among which, the supply function equilibrium (SFE) [6]–[9] and Cournot models [10]–[12] are the most extensively used models for analyzing pool-based power markets. The Cournot model assumes that strategic firms employ quantity strategies: each strategic firm decides its quantity to produce, treating the output level of its competitors as a constant. The Cournot model often suffers from the problem of sensitivity to the specification of market demands. The SFE model is successful in studying oligopoly behavior in the power market, which offers a more realistic view of the power market. The general SFE model was introduced by Klemperer and Meyer [13] and was first applied to power market analysis by Green and Newbery [14].

The Walrasian equilibrium is also called the competition equilibrium, which is used to determine the clearing price and quantity in the power market from the position of the central auctioneer or ISO. The supply-demand model is a partial application of the competition equilibrium theory and the point where supply and demand intersect is commonly regarded as the equilibrium point in the power market. In [15], a single time period decentralized power market clearing model based on the Walrasian equilibrium is presented, which includes reactive power and demand response in addition to the more common framework of generation-side competition for the real power commodity. What’s more, Walrasian auctions are adopted in [16] to trade demand response based on the market clearing scheme of the so called demand response exchange. As far as we know, the researches on competition equilibrium are comparatively fewer when the power network characteristics are considered.

The main contributions of this paper are three aspects: 1) Marginal cost pricing and accounting cost pricing are

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proposed to formulate the Nash equilibrium and competition equilibrium, respectively; 2) the differences and similarities between the Nash equilibrium and competition equilibrium are explained in detail by establishing the model of electric power systems, where the characteristics of the power network are considered; 3) analysis results show that the different form of equilibrium is closely related to the characteristics of the power network and price mechanism.

The organization of the remainder of this paper is as follows: Section II presents the mathematical model of power market systems. Section III compares the Nash equilibrium and competition equilibrium. The calculation of the equilibrium point is introduced in Section IV. Section V presents the case study by applying the IEEE 30-bus system. Section VI concludes the paper.

II. Power Market Systems

The power market, as with other commodity markets, can be described as an economy system with the participants as generation companies, demanders, ISO (auctioneer) and grid corporations. The ISO and grid corporations are considered as an organic whole in this paper.

The auction problem of the power market can be interpreted as a game where each generation company or customer submits bids. Each bid is a quantity bid, the amount of power to be produced. The central auctioneer (ISO) receives the bids, and decides the power flows in order to maximize power market benefit, subject to the equality and inequality constraints of the power network.

As general market participants, we assume that generation companies and demanders are rational and attempt to maximize their profits

$$\max\ c_k p_k - cost_k(p_k) \tag{1}$$

where $k = 1, 2, \cdots, N$ is node number, N is total number of nodes, there is just one generation company or demander at each node; c_k, p_k are bidding price and power of each generation company or demander, and p_k is positive for generation company and negative for demander; $cost_k(p_k)$ denotes production cost or profit of each generation company or demander. The objective of each generation company or demander is to minimize cost, which can be expressed as

$$\min\ cost_k(p_k) - c_k p_k. \tag{2}$$

As a special participant, the auctioneer considers problems from the position of a social welfare worker, so the benefits of the power market are expressed by the sum of profits from all market participants:

$$\max\ \sum_{k=1}^{N} [c_k p_k - \cos t_k(p_k)] \tag{3}$$

The power balance equation used as equality constraints at each node must be met:

$$\begin{aligned} f_k(p_k) = p_k - e_k \sum_{j\in k} (G_{kj} e_j - B_{kj} f_j) \\ - f_k \sum_{j\in k} (G_{kj} f_j + B_{kj} e_j) = 0 \\ Q_k - f_k \sum_{j\in k} (G_{kj} e_j - B_{kj} f_j) \\ + e_k \sum_{j\in k} (G_{kj} f_j + B_{kj} e_j) = 0 \end{aligned} \tag{4}$$

where e, f denote the real and imaginary part of nodal voltage. The inequality constraints are maximum flow limits through each transmission line:

$$P_l - P_l^{\max} \le 0 \tag{5}$$

where $l = 1, 2, \cdots, L$ is the transmission line number, L is the total number of lines. In order to simply describe the problem, other inequality constraints are ignored in this paper.

In the competition of the power market, the costs of generation companies and profits of demanders, which are not open to the public, cannot be known by the auctioneer, so the objective function (3) is substituted by the formula below:

$$\max\ \sum_{k=1}^{N} c_k p_k \quad \text{or} \quad \min\ -\sum_{k=1}^{N} c_k p_k \tag{6}$$

Formulas (1)–(6) consist of the model of the power market system.

A. Marginal Cost Pricing

According to (3)–(6), the augmented Lagrange function, that denotes the costs of the power market system, can be written as:

$$L = -\sum_{k=1}^{N} c_k p_k + \sum_{k=1}^{N} \alpha_k f_k(p_k) + \sum_{l=1}^{L_I} \beta_l (P_l - P_l^{\max}) \tag{7}$$

where α, β are Lagrange multipliers associated with the equality and inequality constraints. In the above Lagrange function, the nodal reactive power balance equation is not necessary for taking p_k as variables. The marginal electricity price at node k can be written as:

$$C_k^N = \frac{\partial L}{\partial p_k} = -(c_k + c_k' p_k) + \alpha_k + \mu_k \quad k = 1, 2, \cdots, N \tag{8}$$

where L_I is the set of active inequality constraints; $c_k' = \partial c_k / \partial p_k$; $\mu_k = \sum_{l=1}^{L_I} \beta_l \frac{\partial P_l}{\partial p_k}$.

B. Accounting Cost Pricing

Accounting cost pricing is different than marginal cost pricing, which counts price making both ends meet by apportioning costs among all market participants. The average cost pricing is used for example in this paper because it is a familiar form of accounting cost pricing.

First, the active power balance equation at node k can be changed as:

$$f_k(p_k) = p_k + \Delta f_k = (1 + \xi_k) p_k \tag{9}$$

where

$$\Delta f_k = -e_k \sum_{j\in k} (G_{kj} e_j - B_{kj} f_j) - f_k \sum_{j\in k} (G_{kj} f_j + B_{kj} e_j),$$

$\xi_k = \Delta f_k / p_k$. The transmission flow limit through line l can also be changed as:

$$P_l - P_l^{\max} = \sum_{k=1}^{N} [\gamma_{lk} p_k + \Delta P_l(p_k)] = \sum_{k=1}^{N} \gamma'_{lk} p_k \tag{10}$$

where $\Delta P_l(p_k)$ is the sum of the remainder terms except the first order in the Taylor series expansion of $P_l - P_l^{\max}$; $\gamma_{lk} = \partial P_l / \partial p_k$; $\gamma'_{lk} = \gamma_{lk} + \sum_{l=1}^{L_I} \Delta P_l(p_k)/p_k$.

So the average costs of the power market at node k can be expressed as:

$$L_k = -c_k p_k + \alpha_k (1 + \xi_k) p_k + \mu'_k p_k \tag{11}$$

where $\mu'_k = \sum_{l=1}^{L_I} \beta_l \gamma'_{lk}$. The average electricity price at node k is defined as:

$$C_k^W = \frac{L_k}{p_k} = -c_k + \alpha_k (1 + \xi_k) + \mu'_k. \tag{12}$$

III. Nash Equilibrium and Competition Equilibrium

A. Nash Equilibrium

In game theory, Nash equilibrium is a kind of equilibrium state in a game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his or her own strategy unilaterally. If each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep their strategies unchanged, then the current set of strategy choices and the corresponding payoffs constitute the Nash equilibrium.

In the power market, players can be regarded as the generation companies, auctioneers and those demanders who will participate in the future bidding. So, all the participants can get the maximal profits in the Nash equilibrium, that is to say, the power market system (expressed by (1)–(6)) has its solution and it's a multi-objective optimization problem. In this paper, the existence and uniqueness of the Nash equilibrium solution are regarded as true.

The Nash equilibrium is achieved by taking the marginal cost pricing as shown in (8) with $C_k^N = 0$ when the auctioneer is required to compute equilibrium prices, because it is one of the first order optimal conditions. So, the nodal price can be written as:

$$c_k = -c'_k p_k + \alpha_k + \mu_k \quad k = 1, 2, \cdots, N \tag{13}$$

When the Cournot model is adopted, the p_k will be regarded as the decision-making variable of the generation company or demander based on the nodal price c_k. So $c'_k = 0$ and

$$c_k = \alpha_k + \mu_k \tag{14}$$

When the supply function equilibrium (SFE) model

$$c_k = a_k p_k + b_k \tag{15}$$

is adopted, the p_k and c_k will be regarded as decision-making variables of the generation company or demander, so:

$$c_k = -a_k p_k + \alpha_k + \mu_k \tag{16}$$

The optimization is made by substituting the above formulas into the objective functions (1) or (2) and the best strategy of the generation companies and demanders can be obtained. Therefore, the Nash equilibrium of the power market can be reached.

B. Competition Equilibrium

The model of competition equilibrium for the general commodity market can be expressed as:

$$F(C) = S(C) - D(C) = 0 \tag{17}$$

$$CF(C) = 0 \tag{18}$$

where F is the excess supply function; $S(C), D(C)$ are the supply and demand functions; C is the market price. It can be seen that supply and demand are in balance from (17), and the market is cleared with no surplus from (18).

It is obvious that total power balance equation is necessary for the competition equilibrium model, which is absent in the model (3)–(6). When the nodal active power equations are added together, the following is obtained:

$$\sum_{k=1}^{N} p_k - P_L = 0 \tag{19}$$

where P_L is network losses. Similar to (9) and (10), the network losses P_L can be apportioned by p_k

$$P_L = \sum_{k=1}^{N} [\eta_k p_k + \Delta P_{Lk}(p_k)] = \sum_{k=1}^{N} \eta'_k p_k \tag{20}$$

where $\eta'_k = \eta_k + \Delta P_{Lk}(p_k)/p_k$; $\Delta P_{Lk}(p_k)$ is the sum of the remainder terms except the first order in the Taylor series expansion of P_L. Formula (19) is changed as:

$$\sum_{k=1}^{N} (1 - \eta'_k) p_k = 0 \tag{21}$$

By substituting (12) into (7), the Lagrange function is changed as:

$$L = \sum_{k=1}^{N} C_k^W p_k. \tag{22}$$

Further

$$L = \sum_{k=1}^{N} \frac{C_k^W}{1 - \eta'_k} (1 - \eta'_k) p_k \tag{23}$$

To introduce

$$p'_k = (1 - \eta'_k) p_k \tag{24}$$

So

$$L = \sum_{k=1}^{N} \frac{C_k^W}{1 - \eta'_k} p'_k; \quad \sum_{k=1}^{N} p'_k = 0 \tag{25}$$

To compare with (17) and (18), it can be seen that the competition equilibrium may be achieved just as:

$$\frac{C_1^W}{1 - \eta'_1} = \frac{C_2^W}{1 - \eta'_2} = \ldots = \frac{C_N^W}{1 - \eta'_N} = C_0. \tag{26}$$

So (25) is changed as:

$$L = C_0 \sum_{k=1}^{N} p'_k = 0$$
$$\sum_{k=1}^{N} p'_k = 0 \tag{27}$$

where C_0 is defined as the system marginal electricity price. When the demanders are not considered as participants, the above formula is:

$$C_0 \sum_{k=1}^{N_G} p'_k = C_0 P_D; \quad \sum_{k=1}^{N_G} p'_k = P_D \tag{28}$$

where N_G is the set of generation companies, P_D denotes load. The above formula is the same with (17) and (18). The Lagrange function from (27) is calculated as:

$$L^W = C_0 \sum_{k=1}^{N} p'_k + \lambda \sum_{k=1}^{N} p'_k. \tag{29}$$

The first order optimal condition with variable p'_k is:

$$C_0 = -\lambda. \tag{30}$$

It is noted that the system marginal electricity price is the Lagrange multiplier associated to the total power balance equation. It can be obtained by (26)

$$C_k^W = -C_0(1 - \eta'_k). \tag{31}$$

By substituting (31) into (12), the following equation for nodal price can be obtained:

$$c_k = \alpha_k(1 + \xi_k) + C_0(1 - \eta'_k) + \mu'_k. \tag{32}$$

The optimization is made by substituting the above formulas into the objective functions (1) or (2) and the maximum profits of the generation companies and demanders can be obtained by selecting p_k or p_k, c_k as the decision-making variable.

C. Market Surplus Allocation Comparison

By introducing the nodal price as shown in (13) into the Lagrange function (7), the following is obtained:

$$L = -\sum_{k=1}^{N} c'_k p_k - \sum_{k=1}^{N} \alpha_k p_k - \sum_{k=1}^{N} \mu_k p_k + \sum_{k=1}^{N} \alpha_k f_k(p_k) + \sum_{l=1}^{L_I} \beta_l (P_l - P_l^{\max}). \tag{33}$$

To bring (9) and (10) into the above formula, the Lagrange function is changed as:

$$L = -\sum_{k=1}^{N} c'_k p_k + \sum_{k=1}^{N} \alpha_k \Delta f_k + \sum_{k=1}^{N} \sum_{l=1}^{L_I} \beta_l \Delta P_l. \tag{34}$$

The above formula shows that there exists surplus according to the marginal cost pricing when the Nash equilibrium is achieved in the power market. The auctioneers and the grid corporations that are monopolies do not regard profit as the purpose in the market transaction, so the surplus must be allocated within the market participants again. Because the market surplus is the nonlinear function of p_k, it is quite difficult to judge the absolute fairness in surplus allocation after market transactions.

There is no market surplus when the counting cost pricing is used and the competition equilibrium is performed. In other words, the surplus is allotted in the market transaction. As shown in (9), (10) and (20), though the network losses are apportioned and congestions are eliminated among market participants according to the average allocation principle, a different kind of counting cost pricing may be used to decide different surplus allocation principles.

D. Differences between Nash and Competition Equilibriums

The Nash equilibrium and optimization in mathematics are achieved with marginal cost pricing due to the fact that (12) is one of the Kun-Tucker conditions.

But according to economic theory, it indicates the market failure under the Nash equilibrium due to the existence of market surplus, and the maximum market efficiency can be obtained just when the competition equilibrium is achieved.

As shown in Fig.1, the line 1 represents the sum $\sum_{k \in N_G} p_k$ of powers for generation companies and line 2 represents the sum $\sum_{k \in N_D} p_k$ of powers for demanders, where N_D is the set of generation companies and demanders.

Fig. 1(a) shows that shadow area that denotes market surplus when the Nash equilibrium is achieved because $\sum_{k \in N_G} p_k - \sum_{k \in N_D} p_k = P_L$. In Fig. 1(b), the lines 1′ and 2′ are the curve of powers for generation companies and demanders corrected with coefficient $1-\eta'_k$, the competition equilibrium is achieved at point O as: $\sum_{k \in N_G} p'_k = \sum_{k \in N_D} p'_k$.

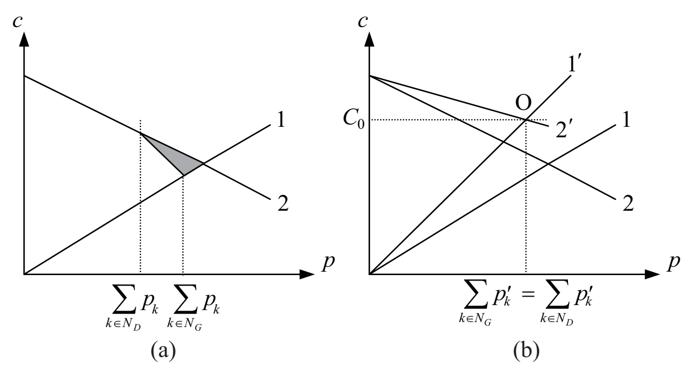

Fig. 1. Power market equilibrium. (a) Nash equilibrium. (b) Competition equilibrium.

Therefore the optimization in economics is realized under the competition equilibrium, which means that market efficiency is maximum and the allocation of market benefits is fair due to the fact that the allocation principle is acting before the market transaction.

E. Similarity between Nash and Competition Equilibrium

If there are no losses and congestions in the power market by disregarding the power network characteristics, the model of the power market system can be simplified as:

$$\begin{cases} \min -\sum_{k=1}^{N} c_k p_k \\ \text{s.t. } \sum_{k=1}^{N} p_k = 0 \end{cases} \tag{35}$$

If the Cournot model is used, the first order optimal conditions for the Nash equilibrium calculation are:

$$c_1 = c_2 = \cdots = c_N = -\lambda. \tag{36}$$

When the supply function model is used, the first order optimal conditions for the Nash equilibrium calculation are:

$$c_1 + a_1 p_1 = c_2 + a_2 p_2 = \cdots = c_N + a_N p_N = -\lambda. \tag{37}$$

Moreover, the conditions for the competition equilibrium calculation are the same with (36). So, there is sameness between the Nash equilibrium and competition equilibrium in the power market when disregarding the power network characteristics.

IV. Calculation of Equilibrium Point

It is a nonlinear programming problem to calculate the equilibrium point, which can be solved by the Newton method or gradient method. The first order optimal conditions for the Nash equilibrium calculation are as follows:

$$\begin{cases} -c_k - c'_k p_k + \alpha_k + \mu_k = 0 \\ p_k - e_k \sum_{j\in k} (G_{kj} e_j - B_{kj} f_j) - f_k \sum_{j\in k} (G_{kj} f_j + B_{kj} e_j) = 0 \\ Q_k - f_k \sum_{j\in k} (G_{kj} e_j - B_{kj} f_j) + e_k \sum_{j\in k} (G_{kj} f_j + B_{kj} e_j) = 0 \\ P_l - P_l^{\max} = 0 \quad l \in L_I \\ k = 1, 2, \cdots, N \end{cases} \tag{38}$$

The calculation of the competition equilibrium point is according to following equations:

$$\begin{cases} -c_k + \alpha_k (1 + \xi_k) + \lambda (1 - \eta'_k) + \mu'_k = 0 \\ p_k - e_k \sum_{j\in k} (G_{kj} e_j - B_{kj} f_j) - f_k \sum_{j\in k} (G_{kj} f_j + B_{kj} e_j) = 0 \\ Q_k - f_k \sum_{j\in k} (G_{kj} e_j - B_{kj} f_j) + e_k \sum_{j\in k} (G_{kj} f_j + B_{kj} e_j) = 0 \\ P_l - P_l^{\max} = 0 \quad l \in L_I \\ \sum_{k=1}^{N} p_k - P_L = 0 \\ k = 1, 2, \cdots, N \end{cases} \tag{39}$$

It differs from (38), and the additional total active power balance equation is required in the above formula. The equal price method [17] can be used to solve the above problem, because there is a uniform system marginal price.

Step 1: To define $m = 0$ and give the initial value of system marginal price $C_0^{(m)}$;

Step 2: Formula (39) is solved according to bidding curves $c_k(p_k)$ in order to determine $p_k^{(m)'}$ for each generator;

Step 3: To judge that equation $\sum_{k=1}^{N_G} p'_k = P_D$ is satisfied or not? If satisfied, the calculating process is ended; otherwise, the system marginal price is corrected with $m = m + 1$ by the following formula:

$$C_0^{(m)} = C_0^{(m-1)} + K\Delta C. \tag{40}$$

And return to Step 2. In (40), K is the step-length and ΔC is the corrected value at each iteration.

V. Case Study

The case study is made on the IEEE-30 system with the nodal data shown in Table I and more information about this system can be obtained in [18]. The active power and reactive power are shown per unit on a basis of 100 MVA.

TABLE I
Data of Nodes

No.	a_k	b_k.	Load Active Power	Load Reactive Power
1	0	0	0.106	0.019
2	350.0	175.0	0.217	0.127
3	0	0	0.024	0.012
4	0	0	0.076	0.016
5	1250.0	100.0	0.942	0.19
6	0	0	0	0
7	0	0	0.228	0.109
8	166.8	325.0	0.3	0.3
9	0	0	0	0
10	0	0	0.058	0.02
11	500	300	0	0
12	0	0	0.112	0.075
13	500.0	300.0	0	0
14	0	0	0.062	0.016
15	0	0	0.082	0.025
16	0	0	0.035	0.018
17	0	0	0.09	0.058
18	0	0	0.032	0.009
19	0	0	0.095	0.034
20	0	0	0.022	0.007
21	0	0	0.175	0.112
22	0	0	0	0
23	0	0	0.032	0.016
24	0	0	0.087	0.067
25	0	0	0	0
26	0	0	0.035	0.023
27	0	0	0	0
28	0	0	0	0
29	0	0	0.024	0.009
30	75.0	200.0	0	0

It is assumed that demanders are not considered as participants and the curves offered by the generation companies are assumed to be a straight line ($c_k = a_k p_k + b_k$) with the unit Yuan/MW.

The gradient method is used to calculate the Nash equilibrium point and the equal price method is used to calculate the competition equilibrium point.

Given the initial value of the system marginal price: $C_0^{(m)} = 300$ Yuan/MW. The step-length and corrected value are set to 1 and 0.001, respectively. The calculating results are listed in Table II with the system marginal electricity price set at 359 Yuan/MW.

It can be seen, there are large differences between the Nash equilibrium and competition equilibrium. The high nodal prices of the Nash equilibrium indicate that a market surplus will be obtained.

TABLE II
CALCULATING RESULTS FOR GENERATORS

Node No.	Nash Equilibrium		Competition Equilibrium	
	Nodal Price (Yuan/MW)	Power (p.u.)	Nodal Price (Yuan/MW)	Power (p.u.)
2	473.668	0.42667	352.656	0.507587
5	632.525	0.21301	356.431	0.205145
8	358.36	0.10000	356.683	0.189949
11	485.213	0.18521	357.577	0.115154
13	488.559	0.18856	357.577	0.115154
30	473.63	1.82420	336.157	1.815430

VI. CONCLUSION

The power market is a typical industrial market with its own characteristics, especially in the power network. By comparing the Nash equilibrium with the competition equilibrium, the following conclusions are obtained:

1) The Nash equilibrium is achieved according to the marginal cost price and the competition equilibrium is performed according to the accounting cost pricing;
2) The surplus appearing in the Nash equilibrium is required to allocate after market transactions, but there is no surplus in the competition equilibrium;
3) The optimization in mathematics is achieved by the Nash equilibrium and the optimization in economics is realized by the competition equilibrium;
4) The Nash equilibrium and competition equilibrium are uniform when the characteristics of the power network are disregarded.

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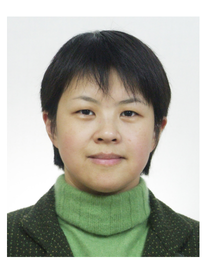

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