Transmission Network Expansion Planning Considering the Generators' Contribution to Uncertainty Accommodation

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***Abstract*—This paper presents an optimization for transmission network expansion planning (TNEP) under uncertainty circumstances. This TNEP model introduces the approach of parameter sets to describe the range that all possible realizations of uncertainties in load and renewable generation can reach. While optimizing the TNEP solution, the output of each generator is modeled as an uncertain variable to linearly respond to changes caused by uncertainties, which is constrained by the extent to which uncertain parameters may change the operational range of generators, and network topology. This paper demonstrates that the robust optimization approach is effective to make the problem with uncertainties tractable by converting it into a deterministic optimization, and with the genetic algorithm, the optimal TNEP solution is derived iteratively. Compared with other robust TNEP results tested on IEEE 24-bus systems, the proposed method produces a least-cost expansion plan without losing robustness. In addition, the contribution that each generator can make to accommodate with every uncertainty is optimally quantified. Effects imposed by different uncertainty levels are analyzed to provide a compromise of the conservativeness of TNEP solutions.**

***Index Terms*—Expansion planning, renewable energy, robust optimization, transmission network, uncertainty.**

NOMENCLATURE

A. Indices and sets

n	Load bus.
i	Generator bus.
j	Renewable energy integrated bus.
mk	Indices of transmission lines between bus m and k.
S_{D}	Set of load bus.
S_{R}	Set of renewable energy integrated bus.
S_G	Set of generators.
$\Re_{\Gamma_{\mathrm{D}}}, \Re_{\Gamma_{\mathrm{R}}}$	Set of variations which are controlled by budget parameters.
Ω	Set of all candidate transmission lines.

Manuscript received November 24, 2015; revised February 22, 2016; accepted May 15, 2016. Date of publication March 30, 2017; date of current version August 2, 2007. This work was supported in part by the National Basic Research Program of China (2012CB215106).

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DOI: 10.17775/CSEEJPES.2015.01190

B. Uncertain parameters

P_{Dn}	Load at bus n
ΔP_{Dn}	Load variation at bus n, i.e. deviation from forecast value
P_{Rj}	Renewable generation at bus j
ΔP_{Rj}	Renewable generation variation at bus j, i.e. deviation from average output
$\Delta P_{\mathrm{Gi}}^{D}, \Delta P_{\mathrm{Gi}}^{R}$	Adjustment provided by generator at bus i to accommodate load and renewable generation variations, respectively.

C. Decision variables

$\overline{P}_{\mathrm{Gi}}$	Output level of generation at bus i.
M_{in}	Dispatch flexibility generator at bus i for load variation at bus n.
T_{ij}	Dispatch flexibility generator at bus i for renewable generation variation at bus j.
ΔP_{Rj}	Renewable generation variation at bus j, i.e. deviation from average output.
α_{mk}	Number of candidate line between bus m and k.
r_n	Load curtailment at bus n.
L	System's voltage stability index.

D. Parameters

$P_{\mathrm{Dn}}^{\min}, P_{\mathrm{Dn}}^{\max}$	Minimum and maximum value of load at bus n.
$\bar{P}_{\mathrm{Dn}}$	Expected value of P_{Dn}, i.e. forecast load at bus n.
$P_{\mathrm{Rj}}^{\min}, P_{\mathrm{Rj}}^{\max}$	Minimum and maximum output of renewable energy at bus j.
$\bar{P}_{\mathrm{Rj}}$	Expected value of P_{Rj}.
$\Gamma_{\mathrm{D}}, \Gamma_{\mathrm{R}}$	Uncertainty budget parameters for load and renewable generations, respectively.
$P_{\mathrm{Gi}}^{\min}, P_{\mathrm{Gi}}^{\max}$	Minimum and maximum output of generator at bus i.
p	Penalty for load curtailment.
η	Weighted coefficients of voltage stability index.
S_{mk}	Element of branch-bus incidence transpose matrix.
$\bar{f}_{mk}$	Power flow limit of a line between bus m and k.
$\alpha_{mk}^{0}, \alpha_{mk}^{\max}$	Initial and maximum number of lines between bus m and k.
$L_{\lim}$	Limit of voltage stability index.

I. Introduction

The transmission network expansion planning (TNEP) problem aims at a least-costing expansion planning scheme in a centralized power system which should serve load reliably [1]. Traditionally, loads forecasted for the future plan have been considered as uncertain elements in TNEP [2]. In recent decades, the trend of delivering power in a low-carbon way has impelled renewable energy technology to develop at a highly rapid pace. This increasing penetration of renewable energy further intensifies the uncertainty in the power system, due to the inherent intermittency and high volatility. Hence, the impacts of renewable energy on TNEP should be investigated to obtain a cost effective design that is also applicable to accommodate both renewable energy and load uncertainties.

Several advanced mathematical approaches have provided means to account for uncertainty in planning TNEP and many other works have been reported in literature [3]–[9]. The main difference lies on the means of describing uncertainty and how to tackle an optimization with uncertainty. Stochastic optimization converts constraints into a probabilistic formulation. For instance, the reliability criterion is formulated as the probability of load curtailment over a specified threshold in [6]. The probabilistic constraints are enforced for a range of scenarios of random variables which are simulated according to the probability distribution function (PDF). This could lead to inaccuracies and less robustness because an exact PDF is hard to obtain in practice and scenario sampling methods cannot ensure full coverage. In contrast, the robust optimization only requires the information of variation range, which is relatively easy to obtain, to model the uncertainty set [10], [11]. Reference [7] was the first paper proposing the concept of robust TNEP design with uncertainties in loads and renewable generations. Relevant works were proposed in [8] and [9] to produce robust TNEP designs with different formulations. Reference [8] employed a Benders decomposition framework to minimize the worst-case curtailment when making a final decision, whereas in [9] the maximum variations of renewable generations and loads were explicitly presented in inequality constraints involved with uncertainties. The application of robust optimization is shown to get rid of the reliance of the PDF assumption, and these proposed models have proven the effectiveness and superiority of robust optimization in dealing with uncertainty.

Other than bringing about the uncertainty concern, the increasing integration of renewable energy also imposes greater requirements on the flexibility of power systems. The flexibility is defined as an ability to respond to variability in [12]. Other than expanding networks to avoid transmission congestion, a major means of flexibility comes from the generators which can adjust their output to accommodate changes in power. Conventional generators play a significant role in harnessing renewable energy in TNEP optimization. This is discussed in [6] with a deterministic TNEP model, and results indicate that the total investment cost is quite different when given different dispatches of generators. In the probabilistic formulation of the TNEP model [6], the output of the generator is considered as a deterministic variable which will be optimized in each scenario of the realization of wind power. As the stochastic optimization approach needs to generate a large amount of scenarios based on the PDF of uncertainties to convert the probabilistic model into a deterministic one to solve, the output of the generators is the scenario-based result. Actually, the output of the generators in the reported robust TNEP models [7], [8] is also considered as a scenario-based deterministic variable. This is because in references [7], [8] several representative scenarios that reflect the worst-case situations are selected and tested during the solving process to ensure the final solution could be robust, i.e. immunized against changes from uncertain parameters in the models. Hence, the output of generators could differ significantly in every considered scenario, because there is not a direct correlation between the output of generators and changes from uncertainties of renewable generation and load.

In this paper, a robust optimization for transmission network expansion planning under uncertainty circumstances is presented. The effects imposed by different uncertainty levels are also analyzed to compromise the conservativeness on TNEP solutions. In particular, the behavior of generators is considered as being able to respond to changes of renewable generation and load power, so that the output of each generator is dealt with as an uncertain parameter limited by its physical operational range. Through quantifying each generator's adjustment with regard to any change caused by uncertain parameters, the formulation proposed in this paper is able to evaluate the contribution that generators can make to accommodating uncertainties during generating the optimal TNEP solution, which could provide insights for decision makers on the required generation flexibility while maintaining load balance. At the same time, this formulation eliminates the assumption of slack bus which is introduced in the TNEP model proposed in [9] to compensate imbalance power. The slack bus in [9] should have a strong ability to deal with all deviations to reach the power and obtain a feasible result, which is not fully practical in real circumstances. Meanwhile, in the power systems with integrated renewable energy, replacing generation from conventional generators with renewables which are weaker in voltage support has led to a higher probability of voltage instability or collapse. The voltage stability assessment is also an essential issue for TNEP optimization [13], [14]. Therefore, this paper introduces a voltage stability constraint based on the well approved static voltage stability index [15], to guarantee a more secure and applicable TNEP scheme.

The remainder of this paper is organized as follows. Section II presents the approach to model uncertainties with parameter sets and how the generators adjust their output to accommodate changes caused by the uncertainties, and proposes the formulation of robust TNEP model. Section III explains the solving approach based on the generic algorithm (GA) and presents the approach to convert a linear problem with uncertainty into a deterministic linear programming by utilizing the theory of robust optimization. The proposed model is tested on the modified IEEE 24 bus-system in Section IV, and conclusions are presented in Section V.

II. Robust Transmission Network Expansion Planning

A. Modelling Uncertainties

TNEP is the comprehensive solution to meet the requirements of load increase and generation expansion planning, and other considerations that may make requests on the capacity of delivering power in transmission networks. Two main factors are considered in this paper. The first is load increase for a future scenario, one of the primary incentives for generation and network expansion. The value of load could however be inaccurate, due to uncertainties in the development of the economy, the forecast approaches that have been utilized and other factors. Another major uncertain element considered is the generation from renewable energy. Intensive integration of renewable energy is one of the ongoing measures to relieve environmental issues. A high penetration rate of renewable energy could provide clean electricity for energy consumptions, but give rise to the concern of uncertainty to power balancing in planning and operational time frames due to their inherent characteristics. Hence the load at bus n, P_{Dn}, and renewable generation at bus j, P_{Rj}, are considered as uncertain parameters to deal with TNEP in a future scenario.

A set with a maximum value, minimum value and expected value is employed to model each uncertain parameter. For load, the expected value $\bar{P}_{\mathrm{Dn}}$ is the forecast value of load at bus n, which can be predicted beforehand from various forecast methods and indicate a high probability of occurrence. The expected value of renewable energy $\bar{P}_{\mathrm{Rj}}$ could take the average output level to represent the expected contribution of renewable generation to load balance. The quantity of $\bar{P}_{\mathrm{Rj}}$ could be the median value of actual output recorded over a long term period. The maximum and minimum values limit the realization of the actual value of the uncertain parameter, i.e. the possible value can't exceed the maximum value or be less than the minimum value. The maximum and minimum values of load, $P_{\mathrm{Dn}}^{\max}$ and $P_{\mathrm{Dn}}^{\min}$, could be expanded by the forecast value $\bar{P}_{\mathrm{Dn}}$ and a prediction error. As for the renewable energy, the minimum value $P_{\mathrm{Rj}}^{\min}$ is obviously taken as zero value. The maximum value $P_{\mathrm{Rj}}^{\max}$ could be the nameplate capacity of renewable energy to cover all possible realizations of generation; this is the most conservative case. In fact, most of the actual output of renewable energy concentrates at a lower level compared with the nameplate value. $P_{\mathrm{Rj}}^{\max}$ can take some representative values that can cover a broad range of output. In this situation, the excess of renewable generation will be possibly curtailed. Hence, the load and renewable generation are allowed to take any value within the limits $\left[P_{\mathrm{Dn}}^{\min}, P_{\mathrm{Dn}}^{\max}\right]$ and $\left[P_{\mathrm{Rj}}^{\min}, P_{\mathrm{Rj}}^{\max}\right]$, respectively.

In this paper, the emphasis is placed on the variations, which are defined as the extent that the actual realization of uncertainty deviates from the expected value:

$$\Delta P_{\mathrm{Dn}} = P_{\mathrm{Dn}} - \bar{P}_{\mathrm{Dn}}, n \in S_{\mathrm{D}} \tag{1}$$

$$\Delta P_{\mathrm{Rj}} = P_{\mathrm{Rj}} - \bar{P}_{\mathrm{Rj}}, j \in S_{\mathrm{R}}. \tag{2}$$

Here ΔP_{Dn} and ΔP_{Rj} are the variations of load and renewable generation respectively. The variations, represented with the Δ-notation, are indeed uncertain parameters and can be modelled with a set, given by:

$$\Delta P_{\mathrm{Dn}} \in \left[\Delta P_{\mathrm{Dn}}^{\min}, \Delta P_{\mathrm{Dn}}^{\max}\right], n \in S_{\mathrm{D}} \tag{3}$$

$$\Delta P_{\mathrm{Rj}} \in \left[\Delta P_{\mathrm{Rj}}^{\min}, \Delta P_{\mathrm{Rj}}^{\max}\right], j \in S_{\mathrm{R}}. \tag{4}$$

The size of new parameter sets is respectively modified by subtracting the corresponding expected values as:

$$\begin{cases} \Delta P_{\mathrm{Dn}}^{\min} = P_{\mathrm{Dn}}^{\min} - \bar{P}_{\mathrm{Dn}} \\ \Delta P_{\mathrm{Dn}}^{\max} = P_{\mathrm{Dn}}^{\max} - \bar{P}_{\mathrm{Dn}} \end{cases}, \quad n \in S_{\mathrm{D}}$$

$$\begin{cases} \Delta P_{\mathrm{Rj}}^{\min} = P_{\mathrm{Rj}}^{\min} - \bar{P}_{\mathrm{Rj}} \\ \Delta P_{\mathrm{Rj}}^{\max} = P_{\mathrm{Rj}}^{\max} - \bar{P}_{\mathrm{Rj}} \end{cases}, \quad j \in S_{\mathrm{R}}$$

and the expected values of ΔP_{Dn} and ΔP_{Rj} are equal to zero.

To deal with variations, a parametric relationship with respect to the variations from load and renewable generation is introduced for each conventional generator in the system. The parametric relationship is described as follows:

$$\Delta P_{\mathrm{Gi}}^{D} = \sum_{n \in S_{\mathrm{D}}} M_{in} \Delta P_{\mathrm{Dn}}, M_{in} \geq 0, i \in S_G \tag{5}$$

$$\Delta P_{\mathrm{Gi}}^{R} = -\sum_{j \in S_{\mathrm{R}}} T_{ij} \Delta P_{\mathrm{Rj}}, T_{ij} \geq 0, i \in S_G \tag{6}$$

where $\Delta P_{\mathrm{Gi}}^{D}$ and $\Delta P_{\mathrm{Gi}}^{R}$ are the respective adjustment provided by a conventional generator i regarding variations ΔP_{Dn} and ΔP_{Rj}; M_{in}, T_{ij} are the allocation coefficients that represents the capability of a conventional generator i to respond to a unit variation in load at bus n and renewable generation at bus j, respectively. Equation (5) provides a constraint that each conventional generator has a capability to adjust its output and follow the variation of its load. Equation (6) shows that the conventional generators will inversely respond to variations from renewable generation; this suggests that there is a transfer of power between conventional generators and renewables, reflecting that the renewable generation is given to ia prioritized dispatch.

The power balance has to be fulfilled, i.e.

$$\sum_{i \in S_G} \Delta P_{\mathrm{Gi}}^{D} = \sum_{n \in S_{\mathrm{D}}} \Delta P_{\mathrm{Dn}}, \text{ and } \sum_{i \in S_G} \Delta P_{\mathrm{Gi}}^{R} = \sum_{n \in S_{\mathrm{D}}} \Delta P_{\mathrm{Rj}}.$$

After incorporation, two constraints limiting the range of allocation coefficients can be obtained:

$$\sum_{i \in S_G} M_{in} = 1, n \in S_{\mathrm{D}} \tag{7}$$

$$\sum_{i \in S_G} T_{ij} = 1, j \in S_{\mathrm{R}}. \tag{8}$$

Constraints (7) and (8) represent that there is a distribution between the contribution that each conventional generator can make to accommodate variations.

With the parametric relationship described by (5)–(8), the capability of adjustment of each conventional generator differs by the optimal value of allocation coefficients M_{in} and T_{ij}. A positive value of the allocation coefficients means that the corresponding generator is active in providing adjustment if a variation occurs. If the value of allocation coefficients is zero, generator i is unavailable to respond to any changes. The parametric relationship quantified by allocation coefficients turns out to share the power imbalance with all available generators. This is more reasonable compared with setting a slack bus that should be strong enough [9]. In addition, the formulation of a parametric relationship maintains the linearity of the model and provides an approach to deal with uncertainties.

B. Robust TNEP Formulation

The aim of a robust TNEP is to obtain a least-cost planning scheme while maximizing the voltage stability margin, under the consideration of variations of load and renewable energy varying within their parameter sets. The formulation is stated based on a dc load as follows:

$$Min\left\{\sum_{mk\in\Omega}c_{mk}\alpha_{mk}+p\sum_{n\in S_{\mathrm{D}}}r_n+\eta\max\{L-L_{\mathrm{lim}},0\}\right\} \tag{9}$$

subject to

$$\begin{aligned}&\Bigg|\sum_{k\in S_G}S_{mk}\left(\bar{P}_{\mathrm{Gk}}+\Delta P_{\mathrm{Gk}}^{D}+\Delta P_{\mathrm{Gk}}^{R}\right)\\&\quad+\sum_{k\in S_{\mathrm{R}}}S_{mk}\left(\bar{P}_{\mathrm{Rk}}+\Delta P_{Rk}\right)\\&\quad-\sum_{k\in S_{\mathrm{D}}}S_{mk}\left(\bar{P}_{\mathrm{Dk}}+\Delta P_{\mathrm{Dk}}\right)+\sum_{k\in S_{\mathrm{D}}}S_{mk}r_k\Bigg|\\&\quad\le\left(\alpha_{mk}^{0}+\alpha_{mk}\right)\bar{f}_{mk}\end{aligned} \tag{10}$$

$$\begin{aligned}&\sum_{i\in S_G}\left(\bar{P}_{\mathrm{Gi}}+\Delta P_{\mathrm{Gi}}^{D}+\Delta P_{\mathrm{Gi}}^{R}\right)+\sum_{j\in S_{\mathrm{R}}}\left(\bar{P}_{\mathrm{Rj}}+\Delta P_{\mathrm{Rj}}\right)\\&\quad+\sum_{n\in S_{\mathrm{D}}}r_n=\sum_{n\in S_{\mathrm{D}}}\left(\bar{P}_{\mathrm{Dn}}+\Delta P_{\mathrm{Dn}}\right)\end{aligned} \tag{11}$$

$$P_{\mathrm{Gi}}^{\min}\le\bar{P}_{\mathrm{Gi}}+\Delta P_{\mathrm{Gi}}^{D}+\Delta P_{\mathrm{Gi}}^{R}\le P_{\mathrm{Gi}}^{\max},i\in S_G \tag{12}$$

$$0\le r_n\le P_{\mathrm{Dn}},n\in S_{\mathrm{D}} \tag{13}$$

$$\alpha_{mk}^{0}\le\alpha_{mk}\le\alpha_{mk}^{\max} \tag{14}$$

$$\alpha_{mk}\in\{0,1\},mk\in\Omega \tag{15}$$

and constraints related to variations ΔP_{Dn} and ΔP_{Rj} (3)–(8).

The objective (9) is composed of the expansion cost of candidate lines in the transmission network, punishment of load curtailment, and the voltage stability index; p and η are the weighted coefficients. Based on the dc power flow model, (10) stands for the branch power flow limit which is represented by injected power at each bus and corresponding coefficients from the branch-bus incidence transpose matrix. It is noted that S_{mk} is affected by the network topology, i.e. it is a function of the result of TNEP. The power balance is ensured in (11). The output of the conventional generator is constrained within its operational range in (12) where $\bar{P}_{\mathrm{Gi}}$ is the output corresponding to the expected value of uncertainties, i.e. $\bar{P}_{\mathrm{Dn}}$ and $\bar{P}_{\mathrm{Rj}}$. This variable can be used in the evaluation of the voltage stability index. Inequality (13) confines the limit of the load curtailment. The constraints (14) and (15) defines requirements of the network expansion variables.

The voltage stability index [15] of the system L is quantified as:

$$L=\max_{n\in S_{\mathrm{D}}}\left\{\left|\sum_{i\in S_G}\frac{Z_{ni}^{*}\dot{S}_i}{\dot{V}_i}\right|\Big/V_n\right\} \tag{16}$$

where $\dot{V}_i$, V_n are the vectors of voltage at the generator bus i and load bus n; $\dot{S}_i$ is the equivalent power at bus I which stems from the other loads of the system; and Z_{ni}^{*} is the complex conjugate of mutual impedances between bus i and n. A smaller value of L represents a larger voltage stability margin. L could be used to check the voltage stability of the TNEP to obtain an optimal result. The voltage stability index could be easily calculated by utilizing an ac power flow model when the result of the TNEP and output of generator $\bar{P}_{\mathrm{Gi}}$ are solved. To guarantee a reliable voltage stability margin, L_{lim} is set as 0.8. Then an estimation of L which is less than L_{lim} is acceptable, or it leads to a punishment to re-optimize the solution.

In the model, as the uncertainty of load and renewable energy are modeled by a parameter set that limits the range of taking values, the optimal TNEP scheme could be accommodative with any realization within the set, i.e. robust to the changes caused by the uncertainty. In addition, this proposed TNEP model considers the contribution of all generators for accommodating uncertainties. The formulation is a mixed-integer nonlinear optimization with uncertainties confined within the given set, which cannot be solved directly using MILP. To make it tractable, Section III presents an approach via the genetic algorithm (GA) and robust optimization is employed to deal with uncertain elements.

III. Solving Approach

A. Solving with a Genetic Algorithm

The nonlinearity of the proposed formulation exists in (10) where the branch-nodal incidence transpose matrix element S_{mk} multiplies corresponding deterministic and uncertain variables. In fact, S_{mk} depends on the network structure. If a TNEP scheme is given beforehand, S_{mk} is parameterized, making the problem easy to be tractable.

In this paper, the GA which has been proven efficient to model integer variables in TNEP problems [16], [17], is adopted in this paper to initialize the network expansion variables. Constraints (14) and (15) are confined when GA updates its population to generate new candidate TNEP schemes.

The optimal solution of TNEP is derived iteratively via GA. The fitness function of GA is (9), the objective of the proposed robust TNEP. It is calculated separately. First, for each population which represents one kind of network expansion planning scheme, the expansion cost is determined after the population is formulated or updated. Then the infeasibility of each population represented by a load curtailment is obtained with a given TNEP scheme through optimization with uncertainty considered. The voltage stability index is finally calculated.

When one kind of candidate TNEP scheme $\{\alpha_{mk},mk\in\Omega\}$ is given by the population of GA, the goal turns out to minimize the load curtailment that represents the infeasibility of the TNEP solution under constraints of (3)–(8) and (10)–(13). As the variations of the load and renewable energy are the main factors affecting the behavior of the generators and expansion needs of the network, the formulation could be rewritten, by substituting (5) and (6) into (10)–(13), comprised of the objective:

$$Min\left\{\sum_{n\in S_{\mathrm{D}}} r_n\right\} \tag{17}$$

subject to (7) and (8), and

$$\begin{aligned}&\left|\sum_{k\in S_{\mathrm{R}}} X_{mk}^{R}\Delta P_{Rk}+\sum_{k\in S_{\mathrm{D}}} X_{mk}^{D}\Delta P_{\mathrm{Dk}}+\sum_{k\in S_G} S_{mk}\bar{P}_{\mathrm{Gk}}\right.\\&\quad\left.+\sum_{k\in S_{\mathrm{D}}} S_{mn}r_k+\sum_{k\in S_{\mathrm{R}}} S_{mk}\bar{P}_{\mathrm{Rk}}-\sum_{k\in S_{\mathrm{D}}} S_{mn}\bar{P}_{\mathrm{Dk}}\right|\\&\le\left(\alpha_{mk}^{0}+\alpha_{mk}\right)\bar{f}_{mk}\end{aligned} \tag{18}$$

$$\sum_{i\in S_G}\bar{P}_{\mathrm{Gi}}+\sum_{j\in S_{\mathrm{R}}}\bar{P}_{\mathrm{Rj}}=\sum_{n\in S_{\mathrm{D}}}\bar{P}_{\mathrm{Dn}}-r_n \tag{19}$$

$$\begin{aligned}&P_{\mathrm{Gi}}^{\min}\le\bar{P}_{\mathrm{Gi}}+\sum_{n\in S_{\mathrm{D}}} M_{in}\Delta P_{\mathrm{Dn}}-\sum_{j\in S_{\mathrm{R}}} T_{ij}\Delta P_{\mathrm{Rj}}\le P_{\mathrm{Gi}}^{\max},\\&\quad i\in S_G\end{aligned} \tag{20}$$

$$0\le r_n\le\bar{P}_{\mathrm{Dn}}+\Delta P_{\mathrm{Dn}}, n\in S_{\mathrm{D}} \tag{21}$$

where

$$X_{mk}^{R}=S_{mk}-\sum_{i\in S_G} S_{mi}T_{ik},\ \text{and}\ X_{mk}^{D}=\sum_{i\in S_G} S_{mi}M_{ik}-S_{mk}.$$

Thus the above formulation is a linear programming and uncertain parameters are only involved in inequality constraints.

As the uncertainties represented by variations of load and renewable energy are limited within the defined parameter sets in (3) and (4), they could be well incorporated within the approach of robust optimization, which has been proved to be an effective means to deal with uncertainty in linear optimization problems [18]. In the idea of robust optimization, the worst-case scenario is considered to have a great impact on the optimal decision. The worst-case scenario refers to that in which uncertainty takes the extreme value of its parameter set. If the decision is accommodative with the worst-case scenarios, it is also accommodative with scenarios in which uncertainty takes any value within the set, i.e. the optimal decision obtained under the worst-case scenario is robust to all realizations of uncertainty within the parameter set. However, the worst-case scenario can also be regarded as being too conservative. A budget parameter is thus introduced to control the size of the parameter set to relax the conservative consideration of uncertainties [11], [19]. Here, two budget parameters are investigated: Γ_{D} is for load and Γ_{R} is for renewable energy. Then parameter sets (3) and (4) are restated as follows:

$$\Re_{\Gamma_{\mathrm{D}}}=\left\{\gamma_n\Delta P_{\mathrm{Dn}}^{\min}\le\Delta P_{\mathrm{Dn}}\le\gamma_n\Delta P_{\mathrm{Dn}}^{\max}, n\in S_{\mathrm{D}}\right\} \tag{22}$$

constrained by $\left\{0\le\gamma_n\le 1,\ \sum_{n\in S_{\mathrm{D}}}\gamma_n\le\Gamma_{\mathrm{D}}\right\}$, and

$$\Re_{\Gamma_{\mathrm{R}}}=\left\{\beta_j\Delta P_{\mathrm{Rj}}^{\min}\le\Delta P_{\mathrm{Rj}}\le\beta_j\Delta P_{\mathrm{Rj}}^{\max}, j\in S_{\mathrm{R}}\right\} \tag{23}$$

constrained by $\left\{0\le\beta_j\le 1,\ \sum_{j\in S_{\mathrm{R}}}\beta_j\le\Gamma_{\mathrm{R}}\right\}$; where γ_n and β_j are the coefficients determined by the choice of Γ_{D} and Γ_{R}, respectively. If the budget parameter reaches the maximum value, namely the number of uncertainty, γ_n and β_j are forced to be 1, meaning that the worst-case scenario is considered in the optimization. If the budget parameter is less than its maximum value, γ_n and β_j will be allocated with a value that is less than 1, reducing the size of the corresponding parameter set of uncertainty.

Formulations (22) and (23) are the limits of variations of load and renewable generation. Actually, according to [19], they are not necessarily limited by the symmetrical distribution to the zero median value. This assumption has a broad range of applications when modeling uncertainties in the real world, especially for generation from renewable energy.

B. Robust Counterpart Formulation

The optimization (17) is linear and can be formulated in the compact form:

$$\begin{aligned}\min\quad&\boldsymbol{cx}\\ s.t.\quad&\boldsymbol{Ax}\le\boldsymbol{b}\\&\boldsymbol{Gx}=\boldsymbol{H}\\&\boldsymbol{l}\le\boldsymbol{x}\le\boldsymbol{u}\end{aligned} \tag{24}$$

where $\boldsymbol{x}\in R^n$ is the vector of the decision variables with upper and lower bounds $\boldsymbol{u},\boldsymbol{l}\in R^n$, and coefficient matrixes $\boldsymbol{c}\in R^n$, $\boldsymbol{b}\in R^m$, $\boldsymbol{A}\in R^{mn}$, $\boldsymbol{G}\in R^{tn}$, and $\boldsymbol{H}\in R^t$. Inequality constraints are comprised of (18), (20), and (21), and equality constraints include (7), (8), and (19). Obviously, the uncertain parameters, i.e. variations of load and renewable energy ΔP_{Dn} and ΔP_{Rj} are only shown in inequality constraints, and equality constraints are related to deterministic variables.

An equivalent linear formulation, namely the robust counterpart, of the inequality constraints with uncertain data modeled by a parameter set could be formulated. Assume a_{ij} is the uncertain paramter; the parameter set of a_{ij} is $[a_{ij}^L, a_{ij}^U]$, the expected value is $\bar{a}_{ij}$. With the budget parameter Γ_i to control the size of the parameter set, the set $[a_{ij}^L, a_{ij}^U]$ could be represented as that in (22). Then the inequality constraint $a_{ij}x_j\le b_i$ could be converted into the counterpart formulation as:

$$a_{ij}x_j\le b_i\Leftrightarrow\begin{cases}\sum_{j=1}^{n}\bar{a}_{ij}x_j+\Gamma_i z_i+\sum_{j\in J_i} p_{ij}\le b_i\\ z_i+p_{ij}\ge\max\left\{(a_{ij}^U-\bar{a}_{ij})x_j,(\bar{a}_{ij}-a_{ij}^L)x_j\right\}\end{cases} \tag{25}$$

where positivevariables z_i and p_{ik} are introduced without physical meaning. This conversion is obtained by utilizting the duality theory and the detailed explanations and proofs can be found in reference [19]. The robust counterpart formulation remains linear, and is able to ammunize against any realization of uncertainty within its parameter set. In addition, the conversion to a robust counterpart will not affect the repersentation of the objective function.

The right part of (20), taken as an example to show the explicit representation of robust counterpart, is then converted into the following form:

$$\begin{cases} \bar{P}_{\mathrm{Gi}} + \Gamma_{\mathrm{R}}\mu_i^R + \sum\limits_{j\in S_{\mathrm{R}}} \xi_{ij}^R + \Gamma_{\mathrm{D}}\mu_i^D + \sum\limits_{n\in S_{\mathrm{D}}} \xi_{in}^D \le P_{\mathrm{Gi}}^{\max}, \\ i \in S_G \\ \mu_i^R + \xi_{ij}^R \ge \max(-T_{ij}\Delta P_{\mathrm{Rj}}^{\max}, -T_{ij}\Delta P_{\mathrm{Rj}}^{\min}),\ j \in S_{\mathrm{R}} \\ \mu_i^D + \xi_{in}^D \ge \max(M_{in}\Delta P_{\mathrm{Dn}}^{\max}, M_{in}\Delta P_{\mathrm{Dn}}^{\min}),\ n \in S_{\mathrm{D}} \\ \mu_i^R, \xi_{ij}^R, \mu_i^D, \xi_{in}^D \ge 0, i \in S_G, j \in S_{\mathrm{R}}, n \in S_{\mathrm{D}} \end{cases} \tag{26}$$

where $\mu_i^R, \xi_{ij}^R, \mu_i^D, \xi_{in}^D$ are newly introduced variables, which in fact are dual variables of constraints of the uncertainty sets (22) and (23). By introducing these variables, the worst-case scenario of variations of load and renewable generation can be quantified in the constraints and can make an impact on the solution. The choice of budget parameters Γ_{R} and Γ_{D} reflects that there is a tradeoff to control the conservatism of uncertainty. The maximum value of budget parameters is the number of uncertain elements, meaning that the maximum variation range is considered. A smaller value means that part of the uncertainties is limited to take value in a relatively narrow range. The complete robust counterpart formulation could be obtained by converting all constraints in (18), (20) and (21) in the same way.

By utilizing the robust optimization approach to deal with uncertain parameters in the inequality constraints, the problem (24) is finally converted into a linear optimization in which all variables are deterministic. Therefore, the final optimization is composed of the objective (17), equality constraints (7), (8) and (19), and the robust counterpart formulation of inequality constraints (18), (20) and (21), and can be solved efficiently.

C. Summary of the Algorithm

The optimal robust TNEP scheme is iteratively derived by applying GA. The solution process is described as follows:

1) Prepare the input data, including the original network, candidate lines, and the generator's operational range. Formulate the uncertainty set by determining the load prediction errors and maximum variations of the renewable generations. Determine the preferred choice of budget parameters.
2) Initialize the parameters of the genetic algorithm, such as the population size, maximum number of iteration, mutation rate and crossover rate of the GA process.
3) Create the initial population which represents a range of candidate planning schemes. For each planning scheme, solve the robust counterpart of (24) to obtain the load curtailment which represents the infeasibility of the solution. The output of the generators corresponding to the expected values of the uncertainties is obtained at the same time, then the voltage stability index is evaluated. The value of the fitness function for each population is finally calculated based on the formulation of (9).
4) Generate the new planning scheme by selection, crossover and mutation operations and then update the population for the next iteration.
5) The process is terminated if the maximum number of the GA iteration is reached. The best population with a least value in the fitness function is regarded as the optimal TNEP scheme. Otherwise, it goes to 4).

It is noted that the GA operation including crossover and mutation is random so that there may exist the same population in different iterations. If each population in each iteration is checked by infeasibility, it can cause a heavy computational burden in solving the robust TNEP optimization. To avoid this situation, a checked candidate set Ψ is introduced. Ψ is used to store all TNEP candidates that have been checked in the GA process. If the new population is already stored in Ψ, the corresponding fitness value can be loaded directly, otherwise, it should be checked and stored in Ψ. This strategy is proven to be effective in solving the proposed robust TNEP model and also save a large number of computational calculations.

IV. Numerical Results

The TNEP optimization accommodating uncertainties by generators is tested on the IEEE 24 bus-system [20]. The solving algorithm based on GA and its robust counterpart is implemented using Matlab R2014a on a PC with Intel Core i5 of 1.8 GHz CPU and 4.0 GB RAM.

The GA parameters are set as follows: 0.01 for the mutation rate and 0.8 for the crossover rate. The population size and the maximum number of iterations are set as 60 and 30, respectively, to overcome the local optimum problem. The checked candidate set Ψ is assumed to have a size that can record all populations.

In the 24 bus test system, there are 34 existing lines and 41 candidates in planning TNEP. Each candidate can be added up to 3 extra lines, making a maximum value of 4 lines in total between two buses. Cost information of the candidates can be found in [21].

For a future condition, the base case data of 17 load buses is expanded to 3 times the original value, which is regarded as the average value of loads. A 5% prediction error is assumed to formulate the load uncertainty set, i.e., the uncertain load variation takes −5%–5% deviation from the forecast data. The penalty coefficients of load curtailment p and voltage stability index η in (9) are set to be 10^5 in order to find the planning scheme without load curtailment and with a reliable voltage stability margin.

All generator's capacities are expanded to 3.3 times for a future consideration and all generators are assumed to have the dispatch flexibility. Two wind sources are integrated into bus 7 and bus 22 with the same capacity of the original conventional generators. Bus 7 is connected to the system with only one line in the existing structure, so bus 7 can be considered as a remote area with a weak link to the load areas. The assumption that wind farms are located at bus 7 can simulate the situation in which the generation from intensive wind farms are merged into one bus and delivered with common transmission lines. Compared with bus 7, bus 22 is connected to a system closed to load areas and other generators. Thus two kinds of wind farm locations are considered in this simulation. The maximum value of the renewable generation is the nameplate capacity; the minimum value is zero; the expected value of the wind power is assumed to be one third of the nameplate capacity. This indicates the most conservative case that renewable generation can take any value within the capacity.

A. Robust TNEP Solutions

The base case is obtained by assuming all uncertainty budget parameters are equal to the maximum value, i.e., $\Gamma_{\mathrm{D}} = 17$ and $\Gamma_{\mathrm{R}} = 2$. The optimal TNEP solution in the base case is as follows: $n_{1-8} = 1$, $n_{2-8} = 2$, $n_{14-23} = 1$, $n_{3-24} = 1$, $n_{6-10} = 2$, $n_{7-8} = 3$, $n_{8-10} = 1$, $n_{10-12} = 2$, $n_{12-13} = 1$, $n_{15-24} = 1$. A total of 15 lines are added in the modified system to expand the transmission capacity to meet the requirement of load increase and generation expansion. The total expansion cost is $5.98 million. As this case represents the most conservative situation (maximum value of budget parameters), the corresponding result is the most robust against all realizations of uncertainties.

The indicator "robustness" is then introduced to estimate the feasibility of TNEP solutions when any realization of uncertain parameters is tested. The following approach is implemented to quantify the robustness of an optimal TNEP solution:

1) Generate scenarios of all uncertainties by implementing the Monte Carlo simulation. The number should be large enough to ensure the reliability of the statistical result.
2) For each scenario, given the TNEP solution that is checked, calculate the quantity of adjustment each generator should provide based on the realization of uncertainties and corresponding allocation coefficients, and then check the violation of constraints (18) and (20). If the actual output of the generator exceeds the operational range or the actual power flow in each line exceeds the transmission capacity limit, it is regarded to be infeasible under this scenario. This is equivalent to employing a dc power flow to check whether there is a curtailment of load and renewable generation. As the requirement of no curtailment is implicitly considered in constraints (5) and (6), a violation on either of the two constraints could represent an infeasible case.
3) After testing all generated scenarios, count the number of the infeasibility. The ratio of infeasibility to the total number of scenarios is the probability of violation. The robustness index is represented by the ratio of feasibility to the total number of scenarios.

Here a common set of parameters for the implementation of the Monte Carlo simulation is presented. The number of sample scenarios that are used to estimate the robustness is assumed to be 16,600 [8]. The prediction error of load is assumed to be normally distributed with a 5% standard deviation. For the realization of renewable generations, the Weibull distribution is first adopted to simulate the wind speed [22], [23] with a scale and shape factor as 8.4 m/s and 1.9622 m/s. Then the simulated wind speed data are used to generate the wind power data according to the wind turbine output relationship:

$$P_w^s = \begin{cases} 0 & 0 \le V^s < V_{ci} \\ P_{\mathrm{rate}}(V^s - V_{ci}) / (V_{\mathrm{rate}} - V_{ci}) & V_{ci} \le V^s < V_{\mathrm{rate}} \\ P_{\mathrm{rate}} & V_{\mathrm{rate}} \le V^s \le V_{co} \\ 0 & V^s > V_{co} \end{cases} \tag{27}$$

where V^s is the simulated wind speed data; P_w^s is the corresponding wind power data. Parameters including cut-in speed V_{ci}, rated speed V_{rate}, cut-out speed V_{co} and nameplate capacity P_{rate} are taken from [5]. The simulated wind power could not exceed the capacity.

As the base case is obtained with the most conservative consideration, then the robustness index is estimated to be 100%, which is shown in Table I.

TABLE I
ROBUST TNEP RESULTS FOR IEEE 24-BUS SYSTEM WITH RENEWABLE ENERGY AND LOAD UNCERTAINTIES

Models	Optimal Scheme	Cost ($, million)	Robustness (%)
Proposed	$n_{1-8} = 1$, $n_{2-8} = 2$, $n_{14-23} = 1$, $n_{3-24} = 1$, $n_{6-10} = 2$, $n_{7-8} = 3$, $n_{8-10} = 1$, $n_{10-12} = 2$, $n_{12-13} = 1$, $n_{15-24} = 1$	5.98	100
TOAT[a] [7]	$n_{1-8} = 1$, $n_{2-8} = 1$, $n_{14-23} = 1$, $n_{3-24} = 1$, $n_{6-10} = 2$, $n_{7-8} = 3$, $n_{8-10} = 1$, $n_{10-12} = 2$, $n_{12-13} = 1$, $n_{15-24} = 1$, $n_{16-17} = 2$	6.37	94.2[b]
BD [8]	$n_{1-8} = 1$, $n_{2-8} = 2$, $n_{14-23} = 1$, $n_{3-24} = 1$, $n_{6-10} = 2$, $n_{7-8} = 3$, $n_{8-10} = 1$, $n_{10-12} = 2$, $n_{12-13} = 1$, $n_{15-24} = 1$, $n_{21-22} = 1$	6.92	100

[a]The optimal scheme that has a higher robustness is presented here for comparison. Other optimal schemes can be found in [7].
[b]The data is quoted from [7], however, it is marked much lower in [8].

In addition, in Table I, the optimal TNEP solution obtained from the proposed method is compared with that from the TOAT method [7] and BD method [8]. The case illustration in [7] and [8] is the same as the assumption adopted in the base case in this paper, so that a comparison can be carried out between the optimal result of the proposed robust TNEP method and the state of art robust TNEP approaches. The robustness index of the two solutions come from the original references.

Obviously, the proposed method is better compared with the recently reported works. The optimal TNEP scheme is the most cost-saving approach without losing robustness. The result from TOAT is not absolutely feasible for all possible values varying within the uncertainty sets. In addition, the total cost is 6.5% higher than that of the proposed method. The BD method produced the planning scheme that holds the 100% percent robustness whereas it costs a 15.7% higher investment in network expansion. The difference between expansion plans obtained from the proposed method, TOAT method and BD method can be seen in Table I, where the latter two methods require 1 or 2 more lines than the proposed method to accommodate uncertainty.

The average computational time of the proposed method is 27 min. For power system planning optimization, the computational performance is acceptable. The convergence characteristics of the proposed algorithm are shown in Fig. 1, with the pre-set population size and maximum iteration. It demonstrates that computational efficiency can be guaranteed within limited generations. In addition, by utilizing the set Ψ, 930 repetitious computations are avoided for infeasibility checks on average.

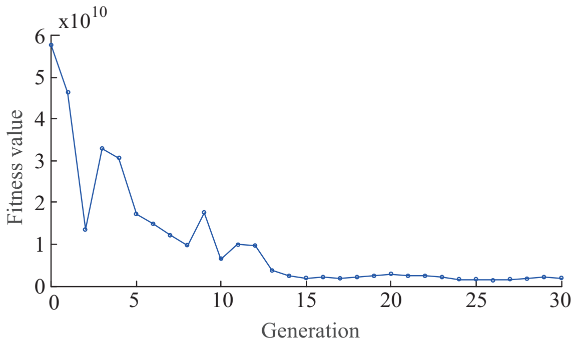


Fig. 1. Fitness curve of the algorithm.

B. Output of Generators and Allocation Coefficients

The proposed method not only produces the cost-saving and robust TNEP scheme but also optimizes the average output level of each generator and quantifies the contribution each generator can deliver to accommodate uncertainties caused by load forecast errors and renewable generations.

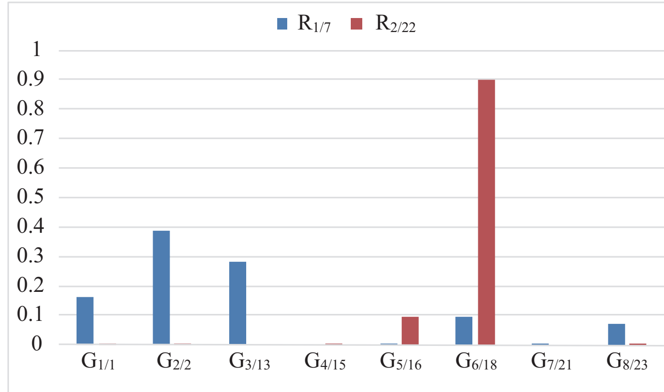
In the base case, the optimal allocation coefficients representing the contribution of generators to accommodating uncertainties are depicted in Fig. 2.

Fig. 2(a) is focused on the allocation coefficients regarding to renewable energy at bus 7 (blue) and bus 22 (red). The quantity is shown with a bar, and the length of bars represents the participation of each generator. G_1, G_2, G_3 are mainly responsible for accommodating with the variations coming from R_1, and G_5 and G_6 are mainly responsible to respond to variations from R_2. G_3 and G_7 do not provide adjustments for either R_1 or R_2.

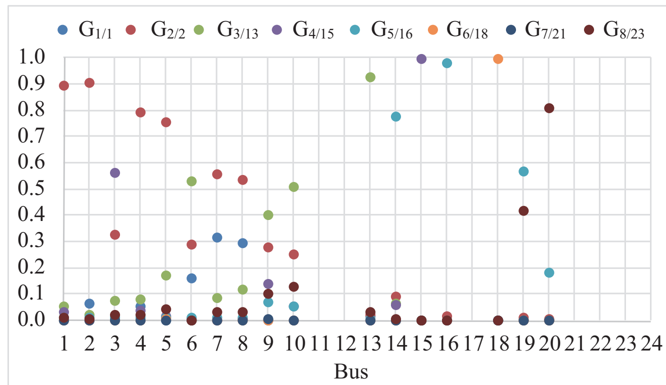
Fig. 2(b) depicts the distribution of allocation coefficients with respect to variations of load. For convenience, all buses in the system are listed in the horizontal axis of Fig. 2(b) where 17 of them are load buses. If the load bus has a generator connected, the generator could be the major source that provides the upward and downward adjustments to variations of the load, for instance, G_2, G_3, G_5, and G_6, or the generators connected at adjacent buses which will respond to changes like G_8.

The optimization of allocation coefficients is bound by several constraints including the extent of variations, operational range of generators, and transmission capacity. So the solution shown in Fig. 2 is a comprehensive result.

The output of generator $P_{\rm Gi}$ is modeled as an uncertain parameter in this model, as it is the sum of the average output $\bar{P}_{\rm Gi}$ corresponding to the expected value of uncertainties, and the adjustment with regard to variations, i.e. $P_{\rm Gi} = \bar{P}_{\rm Gi} + \Delta P^D_{\rm Gi} + \Delta P^R_{\rm Gi}$. The quantity of $\bar{P}_{\rm Gi}$ in the base



(a)



(b)

Fig. 2. The allocation coefficients of each generator with respect to variations of wind power (a) and load (b). Generators and renewable energy are labeled as "Gx/y" and "Rx/y" respectively, where "x" represents the number of generator/renewable energy and "y" denotes the corresponding integrated bus in the IEEE 24-bus system.

case is optimized, and is shown in the third column in Table II. It indicates the behavior of each generator when the average power that renewable energy can generate and the forecast value of load are considered. From Table II, all generators can maintain a relatively high output level when compared with their capacity (listed in the second column).

TABLE II
THE OUTPUT LEVELS OF GENERATORS IN THE BASE CASE (IN MW)

No./Bus	Capacity	Average
$G_{1/1}$	633.6	564.5
$G_{2/2}$	633.6	414.7
$G_{3/13}$	1,950.3	1,780.9
$G_{4/15}$	709.5	638.4
$G_{5/16}$	511.5	420.9
$G_{6/18}$	1,320	830.3
$G_{7/21}$	1,320	1,277.3
$G_{8/23}$	2,178	2,023.0

C. Tradeoff Analysis

To overcome the conservatism of the uncertainty consideration in optimizing TNEP, two strategies are implemented to analyze the tradeoffs for network investment.

1) Reduce the Range of Renewable Generation

In the base case, the maximum value of the renewable generation $P^{\max}_{\rm Rj}$ is assumed to be the capacity value. To relax the conservatism, $P_{\mathrm{Rj}}^{\max}$ is assumed to be 0.7, 0.8 and 0.9 of the capacity, respectively, as the renewable generation can rarely reach the maximum value based on the historical data [24].

Results are shown in Table III. The optimal TNEP solutions are not affected with changes in $P_{\mathrm{Rj}}^{\max}$, so the total expansion costs are the same. The size of the parameter set of variations ΔP_{Rj} is narrowed by decreasing the value of $P_{\mathrm{Rj}}^{\max}$, the solution of $\bar{P}_{\mathrm{Gi}}$ and the corresponding allocation coefficients differ in the three cases. When implementing the estimation of robustness, it is found that the operational range of the generators can cover the actual output of the generators in the three cases, but the power flow of some lines may exceed the transmission capacity. This results in the robustness of the TNEP solution falling down when the value of $P_{\mathrm{Rj}}^{\max}$ decreases.

TABLE III
TNEP SOLUTIONS WITH DIFFERENT CONSIDERATIONS OF THE MAXIMUM RENEWABLE GENERATIONS

Max/Cap	Cost ($, million)	Robustness (%)
1.0	5.98	100
0.9	5.98	96.70
0.8	5.98	89.04
0.7	5.98	88.03

2) Control the Budget Parameters

In the base case, the budget parameter of loads and renewable generations is set as the maximum value, i.e. the number is 19. To demonstrate the effects of budget parameters, the optimization is carried out with the sum of the budget parameters varying from 0 to 19.

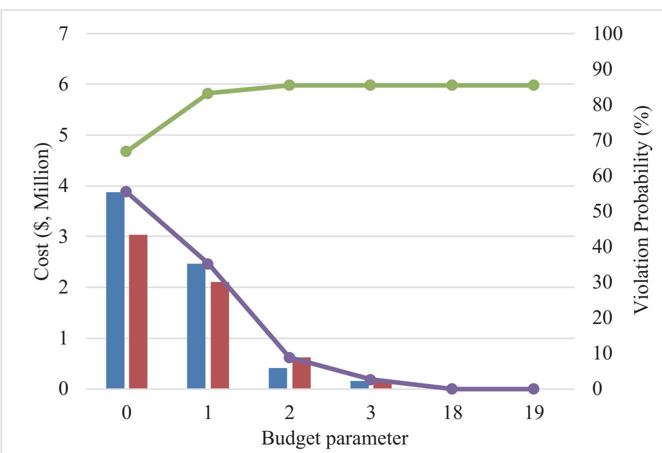


Fig. 3. Expansion cost of TNEP solutions (green line) and corresponding robustness (purple line) versus the choice of budget parameter.

Results of representative cases where the sum of the budget parameters is 0, 1, 2, 3, 18, and 19 are taken and are shown in Fig. 3. Other cases where the sum of the budget parameters take values between 4 to 17 have the same result as that from 18 and 19, which are not shown here.

In Fig. 3, total expansion cost in each case is depicted by the green line, indicating an increase in the cost with the increase of the budget parameters. Remember that the value of the budget parameter controls the size of the uncertainty sets considered in optimization. In the theory of robust optimization, the conservative condition is always considered in the solving process. When the budget parameter is less than the maximum value, those uncertainties which vary in the relatively small parameter sets are prior to being set as deterministic parameters with their expected values. Therefore, cases where the sum of the budget parameters is equal to 1 and 2 are related to uncertainties existing in renewable energy, we let the load buses take the forecast value. From Fig. 3, it indicates that the increase of load is a major element to expand the network, resulting in a total cost of $4.17 million. Then the cost increases to $5.82 million and $5.98 million, implying that uncertainties of renewable generation impose a dominate influence on the increase of expansion costs. It can be seen that the cost remains constant when the budget parameter takes a value more than 2. It means that variations of load could be easily adjusted by generators and will not cause transmission congestion under the network expansion with a total cost of $5.98 million. However, the robustness in each case differs. As the probability of violation represents infeasible conditions, the larger the value of the probability of violation is, the lower the robustness of the TNEP solution is. In Fig. 3, the blue and red bars represent the probability of violation in transmission capacity constraint (18) and (20), and the purple line is drawn with the maximum length of blue and red bars in each budget parameter case which is considered as the probability of infeasibility of the TNEP solution in that case. If no variation is considered for all uncertainties, i.e. the expected values of uncertainties are used in optimizing the TNEP solution, the violation probability could be 55%, which mainly happens in the constraint of transmission capacity (blue bar). Then it falls down with an increase of the value of the budget parameters, indicating an increase in the robustness.

V. CONCLUSION

This paper presented an approach for a centralized system to reach a robust TNEP solution when uncertainties of load and renewable generation are considered. A linear model is put forward in the TNEP optimization to quantify the contributions that each generator can make in response to any changes in every uncertainty by optimizing the allocation coefficients. The approach of robust optimization is introduced in this paper to deal with all uncertain parameters limited within the sets. The estimation of voltage stability is also included to ensure the security of the system.

The increase of renewable energy imposes more concerns of uncertainty and requires more costs on network expansion to ensure a full integration. Compared with previous works on robust TNEP modeling, the proposed method can reach a TNEP solution with costs that are 15.7% lower while maintaining a 100% robustness against uncertainty within an acceptable computational time. The quantified contributions that each generator can make to adjusting its output when changes happen can provide insights to decision makers. In addition, this formulation has shown other compromise TNEP solutions with less conservativeness, while estimating the robustness to provide a reference.

For further research, this static formulation can be extended for a multi-stage outlook period. In the multi-stage formulation, each stage could be presented with a static formulation which is equivalent to the model proposed in this paper, and other constraints related to the sequential relationship between different stages are added. Based on the GA algorithm, the decision variables in each stage can be initialized and updated iteratively to derive the final results. Then the year-by-year expansion on the transmission network could coordinate the yearly increase on demand, generation expansion, and integration of renewable energy.

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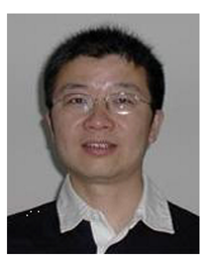

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