Energy Management of Price-maker Community Energy Storage by Stochastic Dynamic Programming

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***Abstract*—In this paper, we propose an analytical stochastic dynamic programming (SDP) algorithm to address the optimal management problem of price-maker community energy storage. As a price-maker, energy storage smooths price differences, thus decreasing energy arbitrage value. However, this price-smoothing effect can result in significant external welfare changes by reducing consumer costs and producer revenues, which is not negligible for the community with energy storage systems. As such, we formulate community storage management as an SDP that aims to maximize both energy arbitrage and community welfare. To incorporate market interaction into the SDP format, we propose a framework that derives partial but sufficient market information to approximate impact of storage operations on market prices. Then we present an analytical SDP algorithm that does not require state discretization. Apart from computational efficiency, another advantage of the analytical algorithm is to guide energy storage to charge/discharge by directly comparing its current marginal value with expected future marginal value. Case studies indicate community-owned energy storage that maximizes both arbitrage and welfare value gains more benefits than storage that maximizes only arbitrage. The proposed algorithm ensures optimality and largely reduces the computational complexity of the standard SDP.**

***Index Terms*—Analytical stochastic dynamic programming, energy management, energy storage, price-maker, social welfare.**

NOMENCLATURE

A. Functions

D/D^{all}	Community/Market load curve function.
f	Probability density distribution.
g_t	Marginal value function starting from time t.
J	Supply curve function.
r_t	Reward function at time t.
V_t	Optimal value function at time t.
W_c/W_d	Community welfare gains during charging/discharging states.

B. Variables

$p_{c,t}/p_{d,t}$	Ex-post charging/discharging price at time t.
u_t/w_t	Charging/Discharging power at time t.
X_t	Storage level at time t.

C. Parameters

a_t/a_t^{all}	Community/Market maximum load at time t.
b/b^{all}	Price elasticity of the community/market load.
C	Energy storage capacity.
d_t/d_t^{all}	Community/Market load at time t.
e_t/h_t	Intercept/Slope of the local linearization of the aggregate supply curve around the forecast market price at time t.
N_{SOC}	Number of discrete intervals of storage levels.
N_{RES}	Number of discrete intervals of renewable generation.
N_{Price}	Number of discrete intervals of market price.
p_t	(Ex-ante) market price before storage operation at time t.
$p_{t,\min}/p_{t,\max}$	Minimum/Maximum electricity price values at time t.
q_t	Renewable generation in the community at time t.
$q_{t,\min}/q_{t,\max}$	Minimum/Maximum renewable energy values at time t.
ρ	The capital cost of the storage.
T	Number of stages.
η_c/η_d	Charging/Discharging efficiency.

I. INTRODUCTION

A. Background and Motivation

LOCATED close to consumers and distributed energy resources, a community energy storage system serves as a buffer to mitigate impact of stochastic energy resources and integrate them into the smart grid [1]. The community

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energy storage system can be a single energy storage system, a group of geographically dispersed energy storage systems coordinated as a virtual power plant, or a deferrable demand cluster. With increasing scale of energy storage [2] and development of local energy market, energy storage is encouraged to participate in the market and play a role in price-setting [3], as announced in market access policies such as the Federal Energy Regulatory Commission's Order 841.

Price-maker community energy storage has a complex interaction with the electricity market. On one hand, it takes advantage of differences in market prices for energy arbitrage. On the other hand, it smoothes price differences by decreasing on-peak loads and increasing off-peak loads, resulting in reduction of arbitrage opportunities. In addition, this price-smoothing effect has a significant impact on social welfare, as it reduces market prices for consumers and profits for producers [4]. This welfare impact is critical for storage-owned communities, as they typically own both consumers and renewable-type producers.

Although energy storage is now allowed to participate in both wholesale energy markets and emerging local energy markets, current bidding and control strategies for energy storage are still under development. Considering the capability of forecasting, decision-making of energy storage may depend on information available up to that time slot, but not on the results of future observations. This is a basic requirement of *non-anticipativity*, which is not fully addressed in current mechanisms, particularly for price-maker storage. Besides, the control strategy of community energy storage will lead to significant welfare changes to the community, which is also inadequately addressed under the non-anticipatory mechanism. Moreover, limited computing power is another challenge for storage participation. Therefore, there is an urgent need to create an efficient bidding and dispatch strategy for price-maker community energy storage.

This paper proposes an analytical stochastic dynamic programming (SDP) algorithm to solve energy management of price-maker community energy storage. Considering the price-maker role and non-anticipativity requirements, the main challenges are (i) characterizing interaction between uncertain environment and storage, (ii) modeling welfare impact in SDP format, and (iii) designing a fast analytical algorithm.

B. Literature Review

Energy management structures for energy storage can be classified into three categories: (i) *one-shot model*, which sets storage dispatch and prices over the entire scheduling period at once, (ii) *two-stage model*, which sets here-and-now decisions before uncertainties and determines wait-and-see dispatch variables after observing uncertainties, (iii) *rolling-window model*, also called multi-stage model, which sets dispatch levels and prices sequentially based on market information forecasts for several future intervals [5]. Extensive literature has investigated the one-shot model. Some neglect environmental uncertainty [6], [7], while others address this issue using stochastic programming, robust optimization [8], etc. While these models are often elegant and easy to solve, they ignore non-anticipativity constraints, relying on hindsight knowledge of future uncertainty realization, or do not fully utilize random information available at that time epoch. Although the two-stage model represents an improvement, it still ignores the non-anticipativity at second stage [9].

When considering various uncertainties in the market, the rolling-window model becomes appealing since non-anticipativity is essential for improving strategies. Two representative methods in rolling-window dispatch are model predictive control (MPC) [10] and SDP [11]. The former solves an open-loop optimal control problem on a finite time window for each sampling time, while the latter derives an explicit feedback law for the entire planning horizon. From the perspective of optimality, SDP conducts optimization over the entire period, making it more suitable for energy storage management with time correlation characteristics. However, the drawback of SDP is it is difficult to solve due to the curse of dimensionality [12]. To overcome this issue, various techniques are proposed, such as analytical solution structures based on duality theory [11]–[14] and policy and value function approximations [15], [16]. The above literature focuses on a price-taker setting. However, there is still a lack of SDP formulation and corresponding effective algorithms designed for a price-maker setting.

Managing community storage in a price-maker setting poses two major modeling challenges, especially under the SDP framework. The first challenge is capturing interaction between storage operations and market environment, which is inherent in the *price-maker setting*. The second challenge is *modeling community welfare*, which represents the external value of storage operations.

The difficulty of price-maker setting is estimating uncertain market information, such as market prices, aggregate supply curves, and market demand. One typical method is to assume perfect knowledge or forecasting of supply-demand profiles [17]. Such an assumption is usually adopted in perfect information games like the Stackelberg game. Although this approach represents interaction between market price and energy storage in sophisticated ways, it requires complete market information sometimes difficult for a single market player to obtain. Another approach is to predict price quota curve (PQC). Construction of a PQC requires partial knowledge of the market, including aggregate supply curves, market prices, and demand [18]. In [19], optimal strategies for a price-maker hydro producer were derived from a set of known PQC scenarios. Work in [19] proposed robust optimization for price-maker energy storage to manage the uncertainty risk associated with forecasted PQCs. These contributions mainly consider impacts of storage operation on price in the one-shot, two-stage, or MPC-type models. However, incorporating the price-maker setting into the SDP framework remains a challenge.

When it comes to community welfare modeling, there is extensive literature maximizing social welfare for a storage-owned system [6], typically from the perspective of a community operator or a market manager. This formulation is intuitive, as operator seeks to maximize social welfare of producers, consumers, and energy storage. Since storage operation naturally affects market price, it further influences social welfare of producers and consumers. However, when considering energy storage dispatch, limited literature attributes welfare changes to energy storage usage. Their storage operations only maximize energy arbitrage and ignore social welfare gains [11]–[16], which do not sufficiently reflect the value of storage use, especially for community-owned storage. While reference [4] provides an insightful study of welfare impacts from storage use, simplified market settings and lack of uncertainty analysis may hinder application of models to practical electricity markets. Furthermore, adapting welfare impacts to the SDP framework is still under development.

In summary, Table I outlines different types of research conducted on storage management, as well as key points discussed in this paper.

TABLE I
TYPES OF RESEARCH CONDUCTED ON OPTIMAL STORAGE MANAGEMENT

Ref.	Dispatch structure	Solution method	Impact on the market price	Market information	Welfare impacts
[6]	One-shot		No	–	No
[17]	One-shot	–	Yes	Complete	No
[18]	One-shot	–	Yes	Partial	No
[9]	Two-stage	–	Yes	Partial	No
[10]	MPC	–	No	–	No
[12]	DP-MPC	–	No	–	No
[11]	SDP	Analytical form	No	–	No
[13]	SDP	Analytical form	No	–	No
[14]	SDP	Analytical form	No	–	No
[15]	SDP	Approximation	No	–	No
[16]	SDP	Approximation	No	–	No
This paper	SDP	Analytical form	Yes	Partial	Yes

Note: If storage management has no impact on market price, market information only requires price, thus using "–" for simplicity. For one-shot, two-stage, or MPC dispatch structures, commercial solvers can be used to solve the problem, thus using "–" in the column for the solution method.

C. Contributions

This paper proposes an analytical SDP algorithm for the energy management of price-maker community energy storage. The main contributions are threefold:

1) This is the first paper that proposes an analytical SDP algorithm for price-maker storage management. We show a rigorous threshold structure of optimal policy. Specifically, by applying KKT conditions, derivative of the future value function can be represented exactly as a piecewise linear function, which avoids state discretization as in standard SDP algorithm, thus accelerating computation speed. The analytical algorithm can guide energy storage to charge/discharge by directly comparing its current marginal value with expected future marginal value.

2) To adapt to the SDP model, we present a simple bidding scheme for price-maker energy storage. Inspired by the PQC method, the scheme requires only partial market information, including real-time price forecast, slopes of aggregate supply curve, and slope of market demand curve. It provides a reasonable balance between fidelity and tractability of the model.

3) We quantitatively characterize impacts of storage operation on social welfare changes and tailor them to the SDP model. Hence, operation strategy of storage maximizes both arbitrage profits and community welfare gains. It enlarges social welfare of the whole community that shares this storage. Such a strategy will help the community more fully assess the value of energy storage.

II. SYSTEM SETUP

A. System Structure

There are three parties in the community: (local) consumers, renewable energy, and energy storage. The community acts as a prosumer participating in the real-time market, which sells/buys power to/from the electric grid at real-time prices. Energy storage has a relatively large scale such that it influences market price. In this study, we focus on valuation and impacts of energy storage operations, rather than economic values of excess generation sales or load cost in the community.

There are mainly two kinds of operational valuation of price-maker energy storage, namely, energy arbitrage gains [20] and community welfare gains [4]. Energy arbitrage takes advantage of price differences by buying and storing energy when prices are low and discharging and reselling it when prices are high. To understand social welfare changes, we have Fig. 1. It shows interaction between energy storage, electricity market, and social welfare. When market price is low, energy storage tends to charge, then the price will rise and flexible demand falls, thus consumer surplus will decrease and producer surplus increases, and vice versa. In Fig. 1, we differentiate the electricity market environment into ex-ante and ex-post stages. Ex-ante and ex-post refer to stages before and after storage operation, respectively.

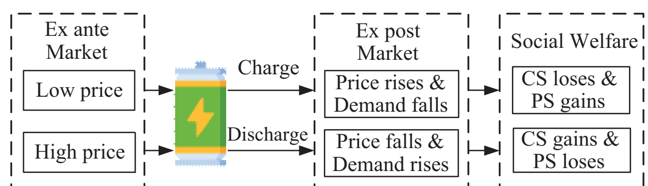

Fig. 1. A general structure of financial impacts among energy storage behaviors, the electricity market, and social welfare. CS is short for consumer surplus, while PS is short for producer surplus.

Assume both renewable energy and storage discharging have zero marginal costs, while storage charging is considered to have highest marginal utility due to its ability to smooth market price. Therefore, for arbitrage profits analysis, there is no difference between considering renewable energy/storage bidding in the market (front-of-meter) and directly supplying load (behind-the-meter). However, this is not the case for welfare analysis. Since energy storage itself has no surplus, if its behaviors are behind the meter, community welfare changes will be miscalculated. Hence, in this paper, we consider renewable, load, and energy storage bid in front of meters, that is, they bid separately in the market.

B. Market Setting

We present essential market elements in the energy storage management problem. The scope of this paper does not include designing predicting techniques for stochastic market information. However, with historical market data available, it is reasonable to assume renewable energy, market price, and local slopes of supply curve are predictable. In this paper, we utilize the ARIMA method. Other advanced forecasting techniques, such as those introduced in [21] and [22], can also be employed.

1) Community Load and Total Market Load

Denote community load by d_t and assume it consists of two parts:

$$d_t = D(p_t) = \begin{cases} a_t - bp_t & p_t \le p_t^{\max} \\ 0 & p_t > p_t^{\max} \end{cases} \tag{1}$$

where p_t is market price and a_t represents community maximum load. If p_t is larger than highest acceptable price level $p_t^{\max}$, loads will decrease to zero. b is related to price elasticity of community load. $a_t, b > 0$. Subscript t refers to time index and will be used throughout the rest of the paper. Similar formulations are applied to total market load d_t^{all} with structure D^{all} and parameters a_t^{all}, b^{all}, $p_t^{\text{all,max}}$.

$$d_t^{\text{all}} = D^{\text{all}}(p_t) = \begin{cases} a_t^{\text{all}} - b^{\text{all}} p_t & p_t \le p_t^{\text{all,max}} \\ 0 & p_t > p_t^{\text{all,max}} \end{cases} \tag{2}$$

2) Stochastic Community Renewable Energy

Define q_t as renewable generation in the community. We model renewable generation as a random process $\{q_t\}$ with the probability density distribution $f(q_t)$. For clarity and simplicity, assume $f(q_t)$, $t = 1, 2, \cdots, T$, are stagewise independent, which is a common assumption in stochastic dual dynamic programming [23]. Note a Markovian form of dependence where q_t depends on q_{t-1} is also applicable in our model.

3) Stochastic Market Price

The intersection of aggregate supply curve and load curve is market price p_t. Similar to community renewable energy, we model market price as a random process $\{p_t\}$ with probability density distribution $f(p_t)$ and $f(p_t)$, $t = 1, 2, \cdots, T$, are stagewise independent.

4) Local Slopes of the Supply Curve

Denote $g_t = J(p_t)$ as aggregate supply curve, where g_t is aggregate generation. Shape of $J(\cdot)$ depends on market structure and generator offers. It can be (approximately) polynomial, piecewise linear, and stepwise [24]. Accurately estimating $J(\cdot)$ is difficult. Fortunately, we do not need to know complete information about $J(\cdot)$. We aim to quantitatively estimate effects of storage charging/discharging behaviors on market prices, thus local information on supply curve around the forecasted market price is enough. An intuitive yet useful idea is to make the first-order Taylor expansion around the forecasted market price $\mathbb{E}p_t$:

$$\begin{aligned} g_t &= J(p_t) \approx J(\mathbb{E}p_t) + J'(\mathbb{E}p_t)(p_t - \mathbb{E}p_t) \\ &\Rightarrow p_t = \mathbb{E}p_t + (g_t - J(\mathbb{E}p_t))/J'(\mathbb{E}p_t) = e_t + h_t g_t \end{aligned} \tag{3}$$

where $\mathbb{E}$ denotes expectation. e_t and h_t are intercept and slope of local linearization of aggregate supply curve around $\mathbb{E}p_t$. We assume $J(\cdot)$ is non-decreasing, thus, $h_t \ge 0$. Our main focus is to obtain information on local slope h_t, which addresses the relationship between a supplier's change in quantity and resulting change in price.

Remark 1 (h_t is approximately constant but e_t is often stochastic). Note in the electricity market, suppliers are mainly traditional generators and RESs. Since marginal prices of traditional units are higher than those of RESs, aggregate supply curve can be divided into two parts: RES-aggregation part and traditional generator-aggregation part. Due to high uncertainty of RES, RES-aggregation part changes greatly. For traditional units, bidding strategies are more stable. Since there are many generators in the market, the strategic change made by a single unit has a relatively small effect on traditional generator-aggregation part. Thus, sensitivity of this part to price can be regarded as unchanged. That is, slope of the local linearized curve h_t is approximately constant [24]. In contrast, intercept of the local linearized curve e_t is stochastic due to shifts of RES-aggregation part.

Based on the proposed system setup and market setting, we present a control scheme of energy storage illustrated in Fig. 2. There are three parts to the scheme: at the beginning, uncertainty forecasting is conducted based on historical data. Then, storage management is presented to derive a lookup table of optimal state-action pairs. After obtaining the lookup table, community operates storage in the real-time market.

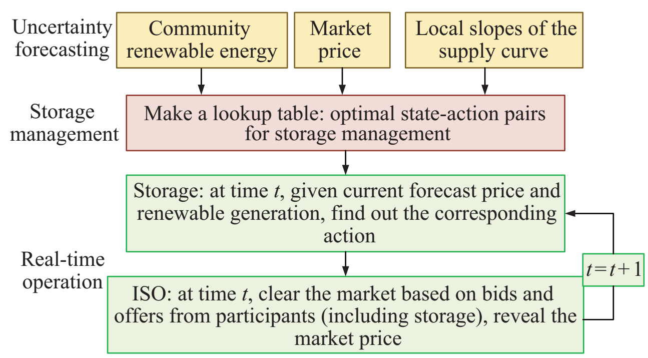

Fig. 2. The control scheme of energy storage in the real-time market.

III. Energy Management of Community Energy Storage

In this section, we first illustrate the energy storage model and analyze community welfare gain, and then formulate the storage management problem as an SDP.

A. Energy Storage Model

The energy storage model can be characterized by the following metrics:

1) Energy capacity: size of storage, denoted by C.

2) Round trip efficiency: ratio of energy discharged to demand to energy charged from the system over each cycle. In this paper, two specific conversion loss efficiencies are used; namely, charging efficiency η_c and discharging efficiency η_d. $\eta_c, \eta_d \in (0, 1)$.

Denote u_t as charging power and w_t as discharging power. Due to conversion losses, $\eta_c^{-1} u_t$ MW of energy should be purchased from the market to ensure u_t MW is stored in storage. Correspondingly, $\eta_d w_t$ MW of energy can be received by the grid if storage discharges w_t MW.

B. Community Welfare Gains

For clarity, we divide community behaviors into three possible states: i) Ex-ante state, ii) Ex-post-charging state and iii) Ex-post-discharging state. We use subscripts c and d to represent charging and discharging states, respectively. Under each state, market equilibrium is intersection of total demand curve and aggregate supply curve.

$$p_t = (D^{\text{all}})^{-1}(d_t^{\text{all}}) \approx e_t + h_t g_t,\ \ g_t = d_t^{\text{all}} - q_t \tag{4}$$

$$\begin{aligned} p_{c,t} &= (D^{\text{all}})^{-1}(d_t^{\text{all}}) \approx e_t + h_t g_{c,t}, \\ g_{c,t} &= d_t^{\text{all}} - q_t + \eta_c^{-1} u_t \end{aligned} \tag{5}$$

$$\begin{aligned} p_{d,t} &= (D^{\text{all}})^{-1}(d_t^{\text{all}}) \approx e_t + h_t g_{d,t}, \\ g_{d,t} &= d_t^{\text{all}} - q_t - \eta_d w_t \end{aligned} \tag{6}$$

where market prices $p_{c,t}$ and $p_{d,t}$ are defined as ex-post charging price and discharging price, respectively. p_t can be regarded as an ex-ante price. According to (4)–(6), expressions of $p_{c,t}$ and $p_{d,t}$ are given as

$$p_{c,t} = p_t + h_t \eta_c^{-1} u_t / (1 + b^{\text{all}} h_t) \tag{7}$$

$$p_{d,t} = p_t - h_t \eta_d w_t / (1 + b^{\text{all}} h_t) \tag{8}$$

According to [4], changes in market prices and total load will result in welfare changes for both consumers and generators in charging and discharging states. Fig. 3 depicts community welfare changes of prosumer-oriented community. Note Fig. 3 only shows when demand is larger than renewable energy. A similar analysis could be conducted when renewable energy plays a dominant role. We are interested in storage effects on community welfare gains $W_c(\cdot)$ and $W_d(\cdot)$ during charging and discharging states:

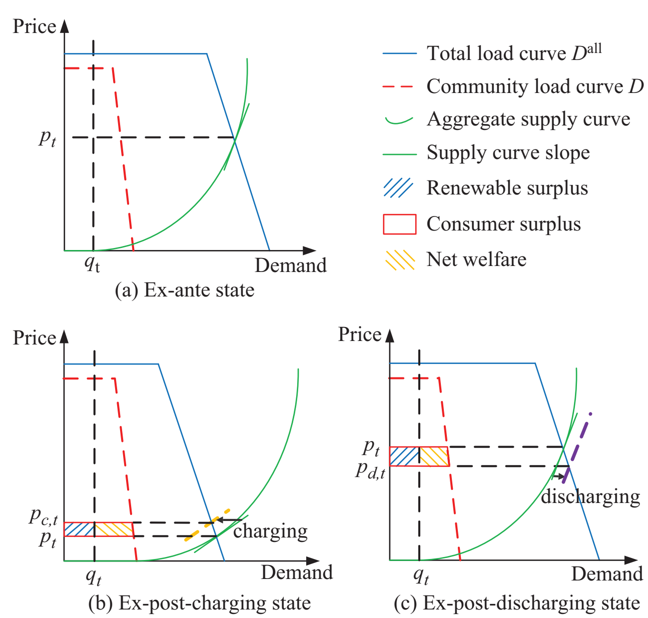

Fig. 3. Ex-ante and ex-post market equilibrium. (a) Ex-ante market equilibrium with community load and renewable bid. (b) Ex-post market equilibrium with community load, renewable, and storage charging bid. (c) Ex-post market equilibrium with community load, renewable, and storage discharging bid. Note that welfare changes due to storage use. The charging behavior of the storage will lift the price, thus decreasing the consumer surplus, but increasing the renewable surplus. Discharging helps lower the price, increase the consumer surplus and decrease the renewable surplus.

$$\begin{aligned} &W_c(u_t|q_t, p_t) \\ &= \underbrace{q_t(p_{c,t} - p_t)}_{\text{Renewable surplus}} - \underbrace{\int_{p_t}^{p_{c,t}} D(p)\mathrm{d}p}_{\text{Community consumer surplus}} \\ &= -(a_t - bp_t - q_t)\frac{h_t \eta_c^{-1}}{1 + b^{\text{all}} h_t} u_t + \frac{1}{2} b \left(\frac{h_t \eta_c^{-1}}{1 + b^{\text{all}} h_t}\right)^2 u_t^2 \end{aligned} \tag{9}$$

$$\begin{aligned} &W_d(w_t|q_t, p_t) \\ &= \underbrace{-q_t(p_t - p_{d,t})}_{\text{Renewable surplus}} + \underbrace{\int_{p_{d,t}}^{p_t} D(p)\mathrm{d}p}_{\text{Community consumer surplus}} \\ &= (a_t - bp_t - q_t)\frac{h_t \eta_d}{1 + b^{\text{all}} h_t} w_t + \frac{1}{2} b \left(\frac{h_t \eta_d}{1 + b^{\text{all}} h_t}\right)^2 w_t^2 \end{aligned} \tag{10}$$

C. Problem Formulation

Energy storage is operated periodically. To avoid plundering opportunity costs from the next period, it is commonly assumed energy storage should return to its initial storage level at the end of the last stage of the current scheduling period:

$$X_{T+1} = X_1 \tag{11}$$

where X is storage level and T is number of stages in a period. Storage level changes with charge and discharge behaviors:

$$X_{t+1} = X_t + u_t - w_t \tag{12}$$

Charging action u_t and discharging action w_t are limited by:

$$0 \le u_t \le C - X_t \tag{13}$$

$$0 \le w_t \le X_t \tag{14}$$

Charging and discharging behaviors of energy storage cannot be conducted simultaneously. Hence, there is a complementary constraint $u_t w_t = 0$, which makes the problem strongly nonconvex. However, if we consider conversion losses, according to [25], the complementary constraint can be eliminated directly without sacrificing feasibility and optimality of the original problem, under the assumption the market price is positive. Intuition is charging and discharging storage simultaneously is not economic if considering conversion loss. Sometimes, negative prices occur in the market, and a simple method is to force storage not to discharge when the price is negative, which is also consistent with many practical market settings [26].

$$w_t = 0 \text{ if } p_t < 0 \tag{15}$$

The storage management problem in real-time market is a rolling-window decision-making problem, which can be characterized as an SDP.

In our SDP format, actions can be expressed in two ways. One is u_t and w_t. The other is $X_{t+1} - X_t$. Those two expressions are interchangeable, since $u_t = X_{t+1} - X_t$ and $w_t = X_t - X_{t+1}$. However, careful selection is necessary for ease of analysis.

System states include 1) endogenous state variable, current storage level X_t, and 2) exogenous stochastic state variables, renewable generation q_t, and market price p_t. Reward function, denoted as r_t, can be divided into two components based on charging and discharging states: $r_{c,t}$ and $r_{d,t}$. These components encompass both arbitrage profits and community welfare gains:

$$r_t(u_t, w_t|q_t, p_t) = r_{d,t}(w_t|q_t, p_t) - r_{c,t}(u_t|q_t, p_t) \tag{16}$$

where

$$\begin{aligned}
&r_{c,t}(u_t|q_t, p_t)\\
&= \underbrace{\eta_c^{-1} u_t p_{c,t}}_{\text{Arbitrage cost}} \ \underbrace{-W_c(u_t|q_t, p_t)}_{\text{Community welfare loss}}\\
&= \left(\eta_c^{-1} p_t + h_t \eta_c^{-1}(a_t - q_t - bp_t)/(1 + b^{\text{all}} h_t)\right) u_t\\
&\quad + \left(\frac{h_t \eta_c^{-2}}{1 + b^{\text{all}} h_t} - \frac{1}{2} b \left(\frac{h_t \eta_c^{-1}}{1 + b^{\text{all}} h_t}\right)^2\right) u_t^2\\
&r_{d,t}(w_t|q_t, p_t)\\
&= \underbrace{\eta_d w_t \cdot p_{d,t}}_{\text{Arbitrage profit}} \ \underbrace{+W_d(w_t|q_t, p_t)}_{\text{Community welfare gain}}\\
&= \left(\eta_d p_t + h_t \eta_d (a_t - q_t - bp_t)/(1 + b^{\text{all}} h_t)\right) w_t\\
&\quad - \left(\frac{h_t \eta_d^2}{1 + b^{\text{all}} h_t} - \frac{1}{2} b \left(\frac{h_t \eta_d}{1 + b^{\text{all}} h_t}\right)^2\right) w_t^2
\end{aligned}$$

After analyzing the above actions, states, and reward function, storage management problem is formulated as follows:

$$\begin{aligned}
&\max_{u_t, w_t} \ \mathbb{E}\left(r_T(\cdot) + \sum_{t=1}^{T-1} r_t(\cdot)\right)\\
&\text{s.t. (11)–(15)}
\end{aligned} \tag{17}$$

Corresponding Bellman equations are given by:

$$\begin{aligned}
V_t(X_{t+1}|X_t, q_t, p_t) &= \underset{u_t, w_t}{\text{maximize}}\, r_{d,t}(\cdot) - r_{c,t}(\cdot)\\
&\quad + \mathbb{E}_{q_{t+1}, p_{t+1}} V_{t+1}(X_{t+1}|q_{t+1}, p_{t+1})\\
V_T(X_{T+1}|X_T, q_T, p_T) &= r_{d,T}(X_T - X_1|q_T, p_T)\\
&\quad - r_{c,T}(X_1 - X_T|q_T, p_T)
\end{aligned} \tag{18}$$

where $V_t(\cdot)$ is optimal value of optimal action X_{t+1} in storage level X_t, renewable state q_t, and market price p_t for a finite-horizon problem starting at time t and ending at time T. $V_T(\cdot)$ is terminal profit.

IV. Analytical SDP Algorithm

SDP models suffer from the "curse of dimensionality", which makes them computationally intractable to obtain optimal solutions; that is, to obtain the optimal solution by backward induction, states need to be highly discretized, in that complexity of backward induction grows exponentially as size of states increases. To tackle this problem, in this section, we show reward function $r_{c,t}(\cdot)$ and $r_{d,t}(\cdot)$ are convex on X_{t+1}, and value function $V_t(\cdot)$ is concave on X_t. Then, by manipulating KKT conditions, future value function can be represented exactly as a piecewise linear function. Hence, optimal non-anticipatory policy is shown to have a threshold structure, which reduces discretization in the backward induction process from V_{t+1} to V_t.

Proposition 1. For every u_t, w_t, and $t = 1, \cdots, T$, $r_{c,t}(u_t|q_t, p_t)$ is convex on u_t; $r_{d,t}(w_t|q_t, p_t)$ is concave on w_t; $r_{c,t}(\cdot)$ and $r_{d,t}(\cdot)$ are convex on X_{t+1}; $V_t(X_{t+1}|X_t, q_t, p_t)$ is concave on X_t under the condition of $\nabla_{X_{t+1}} r_{d,t}(\cdot)|_{X_{t+1}=X_t} \geq \nabla_{X_{t+1}} r_{c,t}(\cdot)|_{X_{t+1}=X_t}$.

Proof: See Appendix for the proof.

Denote $g_t(\cdot) \triangleq \nabla_{X_t} V_t(\cdot)$, which represents marginal value starting from time t. Similarly, $g_{t+1}(\cdot) \triangleq \nabla_{X_{t+1}} V_{t+1}(\cdot)$. In the following Theorem 1, we derive a threshold structure of optimal policy based on Proposition 1. Threshold is to compare $\nabla_{w_t} r_{d,t}$ or $\nabla_{u_t} r_{c,t}$ with $g_{t+1}(\cdot)$ for given X_t, q_t and p_t.

Theorem 1. Threshold structure of the optimal policy for stage t is

$$(w_t^*, u_t^*) = \begin{cases}
(X_t, 0), & \text{if } \nabla_{w_t} r_{d,t}(\cdot)|_{w_t=X_t} > g_{t+1}(\cdot)|_{X_{t+1}=0}\\
(0, C - X_t), & \text{if } \nabla_{u_t} r_{c,t}(\cdot)|_{u_t=C-X_t} < g_{t+1}(\cdot)|_{X_{t+1}=C}\\
(x, 0), & \text{if } \nabla_{w_t} r_{d,t}(\cdot)|_{w_t=X_t} \leq g_{t+1}(\cdot)|_{X_{t+1}=0}\\
& \text{and } \nabla_{w_t} r_{d,t}(\cdot)|_{w_t=0} \geq g_{t+1}(\cdot)|_{X_{t+1}=X_t},\\
& \text{where } x = \sup\{x : \nabla_{w_t} r_{d,t}(\cdot)|_{w_t=x}\\
& \qquad \leq g_{t+1}(\cdot)|_{X_{t+1}=X_t-x}\}\\
(0, y), & \text{if } \nabla_{u_t} r_{c,t}(\cdot)|_{u_t=0} \leq g_{t+1}(\cdot)|_{X_{t+1}=X_t}\\
& \text{and } \nabla_{u_t} r_{c,t}(\cdot)|_{u_t=C-X_t} \geq g_{t+1}(\cdot)|_{X_{t+1}=C},\\
& \text{where } y = \sup\{y : \nabla_{u_t} r_{c,t}(\cdot)|_{u_t=y}\\
& \qquad \leq g_{t+1}(\cdot)|_{X_{t+1}=X_t+y}\}\\
(0, 0), & \text{otherwise.}
\end{cases} \tag{19}$$

Proof: See Appendix for the proof.

According to Theorem 1, optimal control u_t^*, w_t^* depends on balance between expected future marginal value and current marginal value, as shown in Fig. 4. Charging behavior will

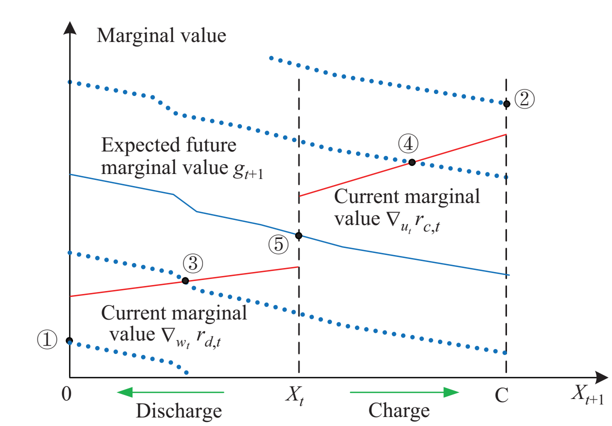

Fig. 4. Expected future marginal value g_{t+1} vs. current marginal value $\nabla_{w_t} r_{d,t}$ or $\nabla_{u_t} r_{c,t}$. Redline segments are current marginal values. Blue lines and dots represent expected future marginal value curves in five possible conditions. The optimal control u_t^*, w_t^* is depicted by black spots and marked as ①②③④⑤, which are illustrated in (19) in sequence.

increase current marginal value but decrease expected future marginal value; discharging is the opposite. Scenario ① tells us if the current marginal value is no less than expected future value, even though energy storage is fully discharged, then energy storage will prefer current profits and thus discharge to empty. In contrast, if current marginal value remains lower than expected future marginal value even in the fully charged situation, storage will charge to full state to obtain enough reserves for future sale, which is Scenario ②. If future marginal value intersects with current marginal value; i.e., in Scenarios ③④, storage will charge/discharge to the storage state indicated by intersection point. Otherwise, there will be no operation under Scenario ⑤.

Corollary 1. The optimal future marginal price for stage t is exactly a piecewise linear function:

$$g_t(\cdot)=\begin{cases}\nabla_{w_t} r_{d,t}(\cdot)|_{w_t=X_t}, \\ \qquad\qquad \text{if } \nabla_{w_t} r_{d,t}(\cdot)|_{w_t=X_t} \geq g_{t+1}(\cdot)|_{X_{t+1}=0} \\ \nabla_{u_t} r_{c,t}(\cdot)|_{u_t=C-X_t}, \\ \qquad\qquad \text{if } \nabla_{u_t} r_{c,t}(\cdot)|_{u_t=C-X_t} \leq g_{t+1}(\cdot)|_{X_{t+1}=C} \\ g_{t+1}(\cdot), \quad \text{otherwise.}\end{cases} \tag{20}$$

Proof: See Appendix for the proof.

Corollary 1 indicates the derivative of the value function $g_t(\cdot)$ is a piecewise linear function of state variable X_t. With Proposition 1, Theorem 1 and Corollary 1, we propose an analytical SDP algorithm to solve the energy management problem. The pseudocode of the analytical SDP is shown in the Algorithm. Storage levels, renewable energy states, and price states are discretized into $N_{\rm SOC}$, $N_{\rm RES}$, $N_{\rm Price}$ intervals, respectively. Minimum and maximum renewable energy values and electricity price values at time t are $q_{t,\min}$, $q_{t,\max}$, $p_{t,\min}$, and $p_{t,\max}$, respectively, which are calculated by point forecast and probability distribution.

Comparing the analytical SDP with the standard SDP, standard SDP needs to discretize the state variable (next storage level X_{t+1}) to construct future value functions. In contrast, analytical SDP avoids the dimensionality problem by representing derivative of future value function as a piecewise linear function exactly (Corollary 1). Therefore, the optimal policy has a threshold structure (Theorem 1), which can be applied to the analytical SDP to largely accelerate the computational speed.

Algorithm 1: Analytical SDP

Input: $X_1 \leftarrow$ const, $q_1 \leftarrow$ const, $p_1 \leftarrow$ const, storage discretization $\{X_{t,0}=0,\cdots,X_{t,N_{\rm SOC}}=C\}$, renewable energy discretization $\{q_{t,0}=q_{t,\min},\cdots,q_{t,N_{\rm RES}}=q_{t,\max}\}$, electricity price discretization $\{p_{t,0}=p_{t,\min},\cdots,p_{t,N_{\rm Price}}=p_{t,\max}\}$, $f(q_t)$, $f(p_t)$

for $t = T$ *to 1* **do**
  for $i =$ *0 to* $N_{\rm SOC}$ **do**
    for $j =$ *0 to* $N_{\rm RES}$ **do**
      for $l =$ *0 to* $N_{\rm Price}$ **do**
        Use Monte Carlo methods to generate training scenarios based on given $q_{t,j}$, $p_{t,l}$, $f(q)$, $f(p)$
        if $t == T$ **then**
          if $X_1 > X_{t,i}$ **then**
            $g_t(\cdot)=\nabla_{u_t} r_{c,t}(\cdot)|_{u_t=X_1-X_{t,i}}$
          else
            $g_t(\cdot)=\nabla_{w_t} r_{d,t}(\cdot)|_{w_t=X_{t,i}-X_1}$
          end
        else
          $g_{t+1}(\cdot)=\mathbb{E}_{q_{t+1},p_{t+1}} g_{t+1}(\cdot)$
          Compute $w_t^*(\cdot), u_t^*(\cdot)$ using (19)
          Compute $g_t(\cdot)$ using (20)
        end
      end
    end
  end
end

V. Case Studies

In this section, we use numerical examples to demonstrate our theoretical results. There are three cases for comparison:

Case 1: regard storage as a price taker.

Case 2: regard storage as a profit-maximizing strategic resource. That is, its objective considers only energy arbitrage.

Case 3: regard storage as a welfare-maximizing strategic resource. That is, the objective includes energy arbitrage and community welfare.

Assume charging/discharging efficiency of the invested storage is 0.9. Rated capacity of energy storage is 20 MWh. Number of intervals of energy storage level is 20. Forecasted real-time prices are chosen from [27] during Sep. 1 to Sep. 24, 2020. Supply curve slopes are based on [28]; we scale them in Table II. Price elasticity for total market load is chosen to be $b^{\rm all}=0.5$. Parameters for community load are set as $a_t=10$, $b=0.2$. Based on the dataset from [29], we take maximum load of 11 residential consumers as community maximum load a_t. Community renewable data is obtained from the 2014 global energy forecasting competition, where wind power can be predicted based on several kinds of weather predictors such as 10-meter U wind, 10-meter V wind, 100-meter U wind, 100-meter V wind, etc. The dataset we select is from Jan. 1st, 2012, to Nov. 30th, 2013 in Zone 1. To match real-time price, we interpolate load and renewable energy into 5-minute granularity. Forecast real-time prices and community renewable production are depicted in Fig. 5. All tests are implemented on a laptop with an Intel Core i5-72000 central processing unit. Optimization problems are solved with MATLAB.

A. Periodicity and Initial Storage Level Settings

Periodicity refers to scheduling period of storage. Initial storage level refers to storage level at the beginning of the cycle.

To choose the most suitable values of periodicity and initial storage level for energy management, we test optimal values of different periodicities under different initial storage levels.

TABLE II
TYPICAL SLOPES OF THE SUPPLY CURVE IN REAL-TIME MARKET

Period	Ex-ante price	Slope	Period	Ex-ante price	Slope
1	0–2	0.004	4	25–38	0.166
2	2–16	0.131	5	38–57	0.665
3	16–25	0.043	6	57–240	6.02

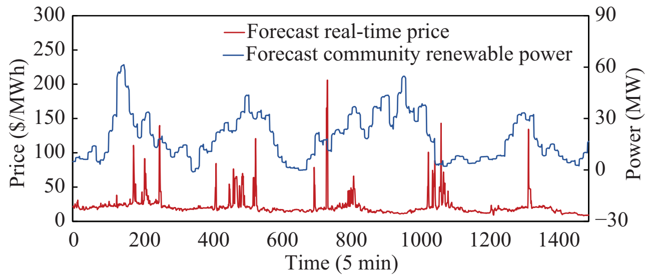

Fig. 5. Forecast real-time prices and community renewable power profile.

Number of sampling points in 1/4 day is 72 since load is collected once every 5 minutes. Finally, periodicity is set as (1, 2, 3, 4, 5, 6) $*72$. That is, periodicity is chosen as 0.25, 0.5, 0.75, 1, 1.25, and 1.5 days.

Figure 6 shows results. Optimal value is calculated as the quarter of the daily profit. For example, if periodicity is selected as one day, we calculate average revenue of one day for the whole planning horizon and divide average revenue by 4 to obtain the quarter of the daily profit. As periodicity changes, optimal value changes largely, because real-time price is volatile. Fig. 6 displays setting one day as the planning horizon gives the best revenue. The reason is our data show in most cases, real-time prices in the first half of the day are low, while prices in the second half are high and volatile; thus, storage tends to charge first and then discharge to earn profits.

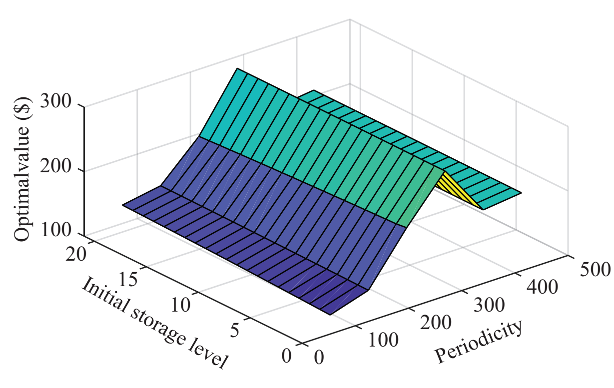

Fig. 6. Optimal values versus periodicity and initial storage levels.

Moreover, in Fig. 6, influence of initial storage levels on optimal value is not significant. Fig. 7 further validates this phenomenon. Although initial storage levels vary from 0 to 20, storage levels become the same from the 3$^{\text{rd}}$ time slot to the 42$^{\text{nd}}$ time slot. The reason is that optimal control decisions become independent of initial levels after several time slots and will become dependent again on initial levels at the end of the operation horizon since the final storage level must be equal to the initial storage level. According to (19), optimal decisions are piecewise linear; hence, mapping from current storage level X_t to the next storage level X_{t+1} is piecewise linear with coefficients not greater than 1. Therefore, initial storage level will only manifestly affect storage levels of the first few time slots.

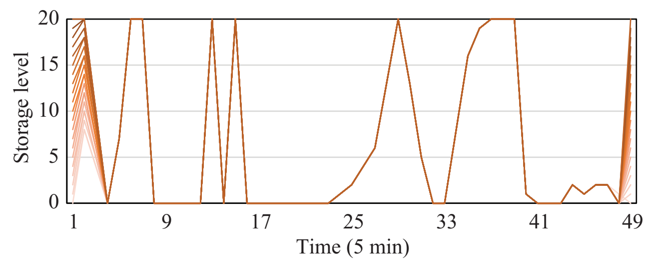

Fig. 7. Storage levels change with different initial storage levels.

In the following discussion, we choose initial storage level as 0 and periodicity as $4*72(*5$ min). Those parameters provide best revenues under the selected data.

B. Price Maker Versus Price Taker

Figure 8 compares optimal values versus capacity in Cases 1, 2, and 3. From Case 1 and Case 2, it can be concluded the price maker gains more value than price taker. This is consistent with our theoretical analysis: as price taker neglects impacts of its operations on market prices, it will misestimate actual income during optimization. Moreover, as storage capacity increases, price-taker assumption becomes even more impractical. In price-maker scenarios, i.e., Case 2 and Case 3, optimizing both energy arbitrage and community welfare brings more benefits than merely considering energy arbitrage. It will help the community more fully assess the value of energy storage.

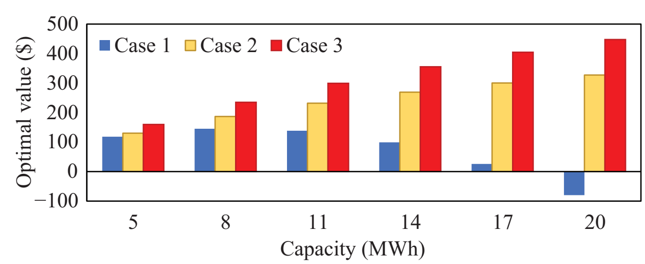

Fig. 8. Optimal values versus capacity under Cases 1, 2 and 3.

C. Energy Arbitrage Gains versus Community Welfare

Figure 9 shows optimal actions under Cases 2 and 3. Different layouts come out in the two cases. In general, storage tends to charge when market price is predicted to be low and discharge when the price is forecasted to be high. From Fig. 10, we can see storage acts as a filter that smooths the price pattern and reduces arbitrage opportunities.

In addition, we analyze arbitrage gains and welfare gains in Cases 2 and 3, respectively, as shown in Fig. 11. Although price-smoothing effects due to storage use reduce arbitrage value, benefits from community welfare enhance value of energy storage. Note even if Case 2 does not optimize community welfare, community welfare still changes because it is a byproduct of arbitrage. Along the period, total value, i.e.,

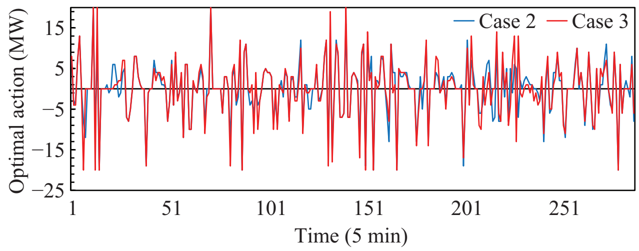

Fig. 9. Optimal actions in Cases 2 and 3. Positive value is charging and negative value means discharging.

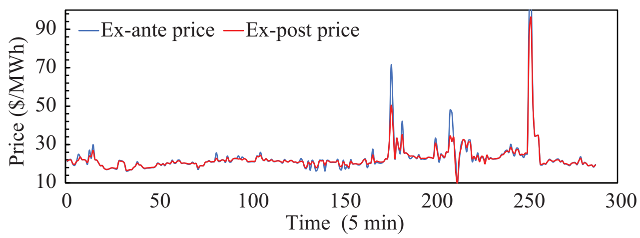

Fig. 10. Market prices change in Case 3, including ex-ante and ex-post prices.

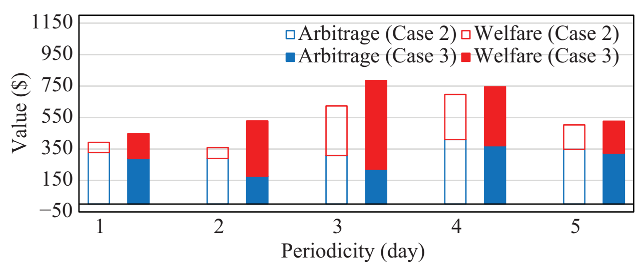

Fig. 11. Optimal values (summation of arbitrage profits and welfare gains) with time variations under Cases 2 and 3.

summation of arbitrage profits and welfare gains, in Case 2 is always smaller than in Case 3 since the objective in Case 3 is exactly total value, yet in Case 2 it is a part of total value (i.e., the arbitrage part). If storage belongs to a merchant, the profit-maximizing purpose in Case 2 is a good choice. However, if storage is owned by prosumers (or consumers/producers), the welfare-maximizing purpose in Case 3 will help gain more benefits, which may exceed 40%.

D. The Proposed Algorithm Versus the Standard SDP

We investigate performance of the proposed algorithm and standard SDP in terms of optimality and computational efficiency under variation of capacity C. Set value of N_{SOC} to 5 C.

Optimality and computational efficiency of the algorithm are illustrated in Fig. 12. The figure shows the proposed analytical SDP achieves the same optimal value as the standard SDP. However, the proposed algorithm is much faster than the standard one. As capacity increases, computation cost of standard SDP increases exponentially, while that of the analytical SDP grows much slower. The computational burden of standard SDP is mainly due to the high granularity of state discretization. While the proposed algorithm avoids state discretization, hence accelerating the execution process.

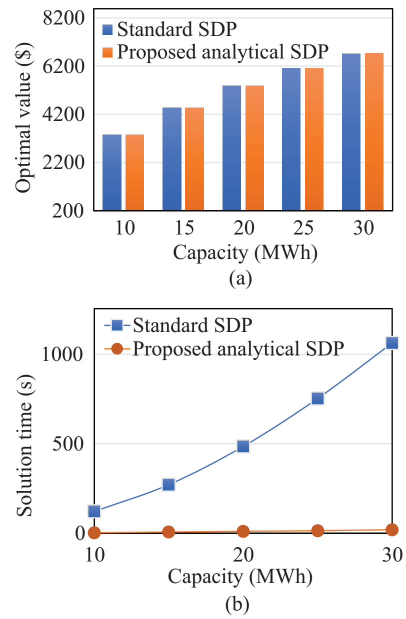

Fig. 12. Comparisons of the proposed algorithm and the SDP algorithm with capacity variations: (a) Optimal value. (b) Solution time.

E. Sensitivity Analysis and Optimal Capacity

Figure 13 shows the change in optimal values with storage capacity and efficiency. We can see as capacity increases, expected profits grow fast at the beginning but tend to be saturated eventually. Furthermore, storage will earn more profits if it has higher efficiencies.

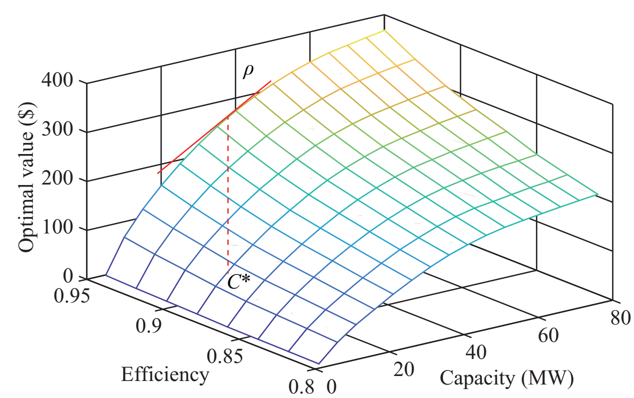

Fig. 13. Optimal values versus storage capacity and efficiency.

In addition, we can derive optimal capacity from Fig. 13. Suppose capital cost of storage is ρ. Adding investment cost to the objective function, we have

$$\underset{C}{\text{maximize}}\, U(C) - \rho C \tag{21}$$

where $U(C)$ is the value-capacity function plotted in Fig. 13 under a given efficiency. Taking the derivative of (21), we have $U'(C^*) = \rho$. Thus, optimal capacity C^* can be obtained.

VI. Conclusion

In this paper, we have proposed an efficient analytical SDP algorithm to solve the price-maker community energy storage. We show the following:

1) The proposed analytical SDP is appealing to price-maker energy storage due to its ease of implementation and efficient computation. The structure has a simple form to make optimal decisions by comparing current marginal value with expected future marginal value. The proposed analytical SDP ensures optimality and reduces complexity of the SDP algorithm from $O(n^2)$ to $O(n)$ level.

2) Arbitrage behaviors of large-scale energy storage will significantly impact market prices and social welfare. Therefore, for prosumer-oriented community energy storage, optimizing both energy arbitrage benefits and community welfare gains becomes crucial, potentially resulting in a 40% increase in profits compared to profit-maximizing approach.

The proposed method can guide prosumer-based communities to utilize energy storage for profit-making. Aside from energy arbitrage and community welfare, participating in auxiliary services accounts for another major source of income for storage, which will be characterized in future work. In addition, techniques for uncertainty modeling of renewable sources and market prices also require further exploration.

Appendix

Proof of Proposition 1 (*Convexity or concavity of current reward function*) Both $r_{c,t}(\cdot)$ and $r_{d,t}(\cdot)$ are quadratic functions; thus, it is easy to analyze the convexity or concavity property. Specifically, we need to deduce the positivity or negativity of the quadratic terms, which is equal to distinguishing the symbol $\pm(1 + b^{\text{all}}h_t 0.5bh_t)$. Note that the slope of the supply curve h_t is nondecreasing; i.e., $h_t \geq 0$, indicating that higher prices make power more profitable to produce. In addition, we have $b^{\text{all}} > b$ by definition. Hence, $r_{c,t}(\cdot)$ is convex on u_t, $r_{d,t}(\cdot)$ is concave on w_t. That is, $\nabla^2_{u_t} r_{c,t}(\cdot) > 0$, $\nabla^2_{w_t} r_{d,t}(\cdot) < 0$.

(*The current reward function is convex on the next storage state*) We have $\nabla^2_{u_t} r_{c,t}(\cdot) > 0$, $\nabla^2_{w_t} r_{d,t}(\cdot) < 0$, and $X_{t+1} = X_t + u_t - w_t$, thus it is easy to prove that $\nabla_{X_{t+1}} r_{c,t}(\cdot)$ and $\nabla_{X_{t+1}} r_{d,t}(\cdot)$ monotonically increase in the X_{t+1}-domain. To obtain that the current reward function increases monotonically for X_{t+1}, we need to figure out the relative size at the boundaries of $\nabla_{X_{t+1}} r_{c,t}(\cdot)$ and $\nabla_{X_{t+1}} r_{d,t}(\cdot)$, that is, to prove $\nabla_{X_{t+1}} r_{d,t}(\cdot)|_{X_{t+1}=X_t} \geq \nabla_{X_{t+1}} r_{c,t}(\cdot)|_{X_{t+1}=X_t}$. It is to determine whether $\eta_c^{-1} p_t + h_t \eta_c^{-1}(a_t - q_t - bp_t)/(1 + b^{\text{all}}h_t)$ is greater than $\eta_d p_t + h_t \eta_d (a_t - q_t - bp_t)/(1 + b^{\text{all}}h_t)$. The determination depends on lots of factors. One main factor is the market price. Since (15) enforces storage not to discharge when the market price is negative. We only need to consider a positive price here. As the renewable energy q_t increases, the slope of the supply curve h_t usually decreases, thus we believe $\nabla_{X_{t+1}} r_{d,t}(\cdot)|_{X_{t+1}=X_t} \geq \nabla_{X_{t+1}} r_{c,t}(\cdot)|_{X_{t+1}=X_t}$ holds.

(*The value function is concave for the current storage state*) Note that $V_t(\cdot) = r_t(\cdot) + \mathbb{E}_{q_{t+1},p_{t+1}} V_{t+1}(\cdot)$. To prove that $V_t(\cdot)$ is concave with for X_t, which means $\nabla^2_{X_t} V_t(\cdot) < 0$, we prove $\nabla^2_{X_t} r_t(\cdot) < 0$ and $\mathbb{E}_{q_{t+1},p_{t+1}}[\nabla^2_{X_t} V_{t+1}(\cdot)] < 0$, respectively.

1) If charging at time t, $\nabla^2_{X_t} r_t(\cdot) = -\nabla^2_{X_t} r_{c,t}(\cdot) = -\nabla^2_{u_t} r_{c,t}(\cdot) < 0$; if discharging, $\nabla^2_{X_t} r_t(\cdot) = \nabla^2_{X_t} r_{d,t}(\cdot) = \nabla^2_{w_t} r_{d,t}(\cdot) < 0$. In summary, $\nabla^2_{X_t} r_t(\cdot) < 0$.

2) Next we use induction to prove $\mathbb{E}_{q_{t+1},p_{t+1}}[\nabla^2_{X_t} V_{t+1}(\cdot)] < 0$. For the terminal profit $V_T(\cdot)$, according to 1), $\nabla^2_{X_T} V_T(\cdot) = \nabla^2_{X_T} r_T(\cdot) < 0$. From $t = T-1$ to 1, $\nabla^2_{X_t} V_{t+1}(\cdot) = \nabla^2_{X_{t+1}} V_{t+1}(\cdot) \times \nabla^2_{X_t} X_{t+1} < 0$. Hence, $\mathbb{E}_{q_{t+1},p_{t+1}} \nabla^2_{X_t} V_{t+1}(\cdot)$, the weighted mean of $\nabla^2_{X_t} V_{t+1}(\cdot)$, is also less than zero. 1) and 2) conclude the proof.

Proof of Theorem 1 The value function of the energy management problem with complete dual variables is shown below

$$
\begin{array}{lll}
V_t(\cdot) = \text{maximize}_{u_t,w_t} & r_{d,t}(\cdot) - r_{c,t}(\cdot) + \mathbb{E}V_{t+1}(\cdot) & \\
\quad \text{subject to} & & \\
\quad \psi: & X_{T+1} = X_1 & \\
\quad (\underline{\mu}_t, \overline{\mu}_t): & 0 \leq u_t \leq C - X_t, \ \forall t & \\
\quad (\underline{\xi}_t, \overline{\xi}_t): & 0 \leq w_t \leq X_t, \ \forall t & \\
\quad \lambda_t: & X_{t+1} = X_t + u_t - w_t, \ \forall t &
\end{array} \tag{A1}
$$

Remember that $g_t(\cdot) = \nabla_{X_t} V_t(\cdot)$ and $g_{t+1}(\cdot) = \nabla_{X_{t+1}} V_{t+1}(\cdot)$. Optimal solutions of the problem (A1) satisfy KKT conditions:

$$-\nabla_{u_t} r_{c,t}(\cdot) - \underline{\mu}_t^* + \overline{\mu}_t^* - \lambda_t^* = 0, \ \forall t \tag{A2}$$

$$\nabla_{w_t} r_{d,t}(\cdot) - \underline{\xi}_t^* + \overline{\xi}_t^* + \lambda_t^* = 0, \ \forall t \tag{A3}$$

$$g_{t+1}(\cdot) + \lambda_t^* = 0, \ \forall t \tag{A4}$$

$$0 \geq \underline{\mu}_t^* \perp \overline{\mu}_t^* \leq 0, \ \forall t \tag{A5}$$

$$0 \geq \underline{\xi}_t^* \perp \overline{\xi}_t^* \leq 0, \ \forall t \tag{A6}$$

$$0 \leq u_t^* \leq C - X_t, \ \forall t \tag{A7}$$

$$0 \leq w_t^* \leq X_t, \ \forall t \tag{A8}$$

$$X_{t+1} = X_t + u_t^* - w_t^*, \ \forall t \tag{A9}$$

According to the five scenarios shown in Fig. 4, we analyze the corresponding optimal solutions.

1) If $\nabla_{w_t} r_{d,t}(\cdot)|_{w_t=X_t} > g_{t+1}(\cdot)|_{X_{t+1}=0}$, according to (A4) and (A3), $\underline{\xi}_t^* - \overline{\xi}_t^* > 0$, thus $w_t^* = X_t$;

2) If $\nabla_{u_t} r_{c,t}(\cdot)|_{u_t=C-X_t} < g_{t+1}(\cdot)|_{X_{t+1}=C}$, according to (A4) and (A2), $\underline{\mu}_t^* - \overline{\mu}_t^* > 0$, thus $u_t^* = C - X_t$;

3) If $\nabla_{w_t} r_{d,t}(\cdot)|_{w_t=X_t} \leq g_{t+1}(\cdot)|_{X_{t+1}=0}$ and $\nabla_{w_t} r_{d,t}(\cdot)|_{w_t=0} \geq g_{t+1}(\cdot)|_{X_{t+1}=X_t}$, then $\underline{\xi}_t^* - \overline{\xi}_t^* \leq 0$ and $\underline{\xi}_t^* - \overline{\xi}_t^* \geq 0$, thus $w_t^* = \sup\{x : \nabla_{w_t} r_{d,t}(\cdot)|_{w_t=x} \leq g_{t+1}(\cdot)|_{X_{t+1}=X_t-x}\}$;

4) If $\nabla_{u_t} r_{c,t}(\cdot)|_{u_t=0} \leq g_{t+1}(\cdot)|_{X_{t+1}=X_t}$ and $\nabla_{u_t} r_{c,t}(\cdot)|_{u_t=C-X_t} \geq g_{t+1}(\cdot)|_{X_{t+1}=C}$, then $\underline{\mu}_t^* - \overline{\mu}_t^* \geq 0$ and $\underline{\mu}_t^* - \overline{\mu}_t^* \leq 0$, thus $u_t^* = \sup\{y : \nabla_{u_t} r_{c,t}(\cdot)|_{u_t=y} \leq g_{t+1}(\cdot)|_{X_{t+1}=X_t+y}\}$;

5) If $\nabla_{w_t} r_{d,t}(\cdot)|_{w_t=0} < g_{t+1}(\cdot)|_{X_{t+1}=X_t}$ and $\nabla_{u_t} r_{c,t}(\cdot)|_{u_t=0} > g_{t+1}(\cdot)|_{X_{t+1}=X_t}$, then $\underline{\xi}_t^* - \overline{\xi}_t^* < 0$ and $\underline{\mu}_t^* - \overline{\mu}_t^* < 0$, i.e., $\underline{\xi}_t^* < 0$ and $\underline{\mu}_t^* < 0$, thus $w_t^* = 0$ and $u_t^* = 0$.

Based on Proposition 1, we know that for X_{t+1}, $r_{c,t}(\cdot)$ and $r_{d,t}(\cdot)$ are convex, and $V_{t+1}(\cdot)$ is concave. Hence, there is at most one intersection of $\{\nabla_{w_t} r_{d,t}(\cdot), \nabla_{u_t} r_{c,t}(\cdot)\}$ and $g_{t+1}(\cdot)$, showing that the storage cannot charge and discharge simultaneously. And we have Theorem 1.

Proof of Corollary 1 We make a sensitivity analysis of $V_t(\cdot)$ to X_t:

$$\nabla_{X_t} V_t(\cdot) = g_t(\cdot) = \overline{\mu}_t^* - \overline{\xi}_t^* - \lambda_t^* \tag{A10}$$

Substituting (A4) into (A10), we have $g_t(\cdot) = g_{t+1}(\cdot) + \overline{\mu}_t^* - \overline{\xi}_t^*$. The following three conditions hold:

1) If $\overline{\mu}_t^* = 0$ and $\overline{\xi}_t^* = 0$, then $g_t(\cdot) = g_{t+1}(\cdot)$;

2) If $\overline{\mu}_t^* \neq 0$ and $\overline{\xi}_t^* = 0$, then $u_t^* = C - X_t$ and $g_t(\cdot) = g_{t+1}(\cdot) + \overline{\mu}_t^* = \nabla_{u_t} r_{c,t}(\cdot)|_{u_t = C - X_t}$;

3) If $\overline{\mu}_t^* = 0$ and $\overline{\xi}_t^* \neq 0$, then $w_t^* = X_t$ and $g_t(\cdot) = g_{t+1}(\cdot) - \overline{\xi}_t^* = \nabla_{w_t} r_{d,t}(\cdot)|_{w_t = X_t}$.

We have Corollary 1.

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