Community-detection-based Approaches for Distribution Network Partition

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***Abstract*—A rational partition is the key prerequisite for the application of distributed algorithms in distribution networks. This paper proposes community-detection-based approaches to a distribution network partition, including a non-overlapping partition and a border-node partitioning method. First, a novel electrical distance is defined to quantify the coupling relationships between buses and it is further used as the edge weight in a transformed equivalent graph. Then, a vertex/link partition community detection approach is applied to over-partition the network into high intra-cohesive and low inter-coupled subregions. Following this, a greedy algorithm and a tabu search method are combined to merge these subregions into target numbers according to the scale similarity principle. The proposed approaches take the influence of three-phase imbalance into consideration and they are decoupled from the power flow. Finally, the approaches are tested on an IEEE 123-bus distribution system and the results verify the effectiveness and the credibility of our proposed methods.**

***Index Terms*—Community detection, distribution network partition, three-phase imbalance.**

I. Introduction

WITH the increasing penetration of distributed energy resources (DERs), as well as a growing number of measurement and control devices deployed, largescale distribution networks are becoming much more complicated and are faced with computational efficiency challenges. Thus, distributed algorithms are introduced into distribution network control, optimization and state estimation (SE) [1]–[5], and are expected to be the main methods implemented in the future.

As an important part of distributed approaches in distribution networks, such as the multi-area state estimation (MASE) and distributed control, a rational network partition is a key prerequisite for ensuring the results and improving calculation accuracy. Generally, each subregion after the partition has an independent control center, which is responsible for SE, control, optimization, and communication with other subregion control centers [6].

The present network partition pattern can be categorized into two main types according to the overlapping properties, non-overlapping partition and overlapping partition [7]. The latter can be further divided into node-overlapping, tie-line overlapping and extended overlapping patterns. In order to reduce the communication time cost and improve the calculation efficiency, the current major partitioning methods are non-overlapping, node overlapping, and tie-line overlapping partitions.

The existing partitioning methods in literature include those based on system topology and geographic location, based on measurement devices and multi-agent system deployment locations, based on electrical distance [8], etc.

Muscas [9] proposes a simple subjective partitioning method based on system topology and geographic location for MASE in a distribution network, which is partitioned into border-node overlapping subregions and each subregion has a similar number of nodes. Similarly, Wei [10] partitions the system into non-overlapping subregions based on similar criteria for MASE, and expands it to the overlapping type. However, such simple subjective partitioning methods lack strict theoretical derivation.

Some partitioning methods quantify the criteria, such as scale similarity and provide optimization solutions. An active distribution network partition model is established from the perspective of measurement configuration and parallel computing efficiency in [11]. Yuan [12] suggests a network partition approach for three-phase MASE based on topology analysis and a postorder-traversal algorithm. However, these methods neglect the influence of coupling relationships between buses.

Additionally, community detection theory is introduced into the power grid community identification method, providing new ideas for the power system partition research area [13]–[15]. Xu [16] proposes a non-overlapping partitioning method based on the improved discrete particle swarm optimization algorithm and community detection theory for distributed reactive power control. But it focuses on the transmission networks and does not consider the three-phase imbalance. Furthermore, the over-partition problems of community detection algorithms have remained unsolved.

Li [17] proposes a partitioning method based on reactive-power-injection/voltage (Q/V) sensitivity [15] and the AP clustering algorithm [19], focusing on distributed reactive power control of active distribution networks. But it relies

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on the power flow and lacks the consideration of three-phase imbalance, that's why it cannot be directly applied in distribution networks where the power flows are in dynamic variation.

We consider the three-phase imbalance in our partitioning method because it is targeted for unbalanced three-phase distribution networks instead of the extreme cases of balanced distribution networks and transmission networks. We are looking forward to providing a more realistic test scenario for three-phase distributed algorithms. While three-phase imbalance is a significant feature of distribution networks, it is not fully considered in the existing partitioning methods, especially those that are sensitivity-based which are defined as being related to the voltage magnitudes [15], [17]. That is to say, the partition pattern is decided by the selected operation situation and the three-phase imbalance actually will change the partition results. Taking the original three-phase imbalanced IEEE-123 system in [18] as an example and partitioning it via the method proposed in [17], it becomes clear that each bus itself is a subregion. Therefore, it is not so persuasive to support the tests of three-phase distributed algorithms in the multi-area distribution network partitioned on the assumption of ideal three-phase balance.

To sum it up, though much research work has been done in the distribution network partition area, the following major shortcomings still exist: 1) The partitioning methods for different application scenarios, such as MASE and distributed control, are completely separated and the factors considered are unilateral. However, the partition results should be equally applicable to these scenarios, for they are implemented by the same local control center. 2) The Q/V or P/V sensitivity based partitioning methods depend on the power flows selected for the calculation, which in fact are in dynamic changes. Thus, it is unavoidable that a very different partition result will be generated if the power flow values change even to a small degree. 3) The three-phase imbalance is not fully considered. 4) The electrical coupling relationship between buses are neglected in the partitioning methods for MASE.

In light of the considerations above, this paper proposes a novel approach to distribution network partition, including the border-node overlapping partition and the non-overlapping partitioning method. By partitioning distribution systems into high intra-cohesive and low inter-coupled subregions with similar scales, it is applicable for different distributed algorithms in large scale distribution networks requiring information exchange, such as MASE and distributed control. It can greatly improve the computation efficiency. One reason is that the subregions' similar scale is beneficial to reduce the communication time cost and thus to improve the distributed computation speed because it means less waiting time for communications between adjacent subregions. The other reason is that the high intra-cohesive and low inter-coupled property is an important factor impacting the computation efficiency. According to [17], this property can help diminish the effect of the operation state variations inside one subregion to the other subregions in distributed control scenarios, resulting in less expected information exchange. It should be noted that the proposed method is based on the assumption that the measuring devices in the system are evenly distributed and that optimal deployment has been achieved. Additionally, when applying the border-node overlapping partitioning method in MASE, it is required that at least one full measurement point is deployed in the overlapping area, while the non-overlapping partitioning method for MASE with the non-overlapping pattern requires measurements of the branch on the tie-line.

The proposed partition approach differs from existing ones in that it has the following contributions. 1) It considers the three-phase imbalance of distribution systems and gives a new definition of electrical distance, which is decoupled with the power flow and only relies on network parameters. 2) The proposed partitioning method considers both the coupling relationship of buses and the size similarity of subregions based on community detection, greedy algorithm and tabu search. 3) With strong theoretical and algorithm support, both the non-overlapping partition and border-node overlapping partitioning method are given in this paper and are equivalently appliable for different scenarios, such as MASE and distributed control. According to the test result in the IEEE 123-bus system, the proposed partition approach is proved to be effective and credible.

The remainder of this paper is organized as follows. Section II introduces the proposed approach to non-overlapping partition from 4 perspectives, the framework, the definition of electrical distance, the community-detection-based initial partition, and the subregion merging. The proposed approach to the border-node overlapping partition is presented in Section III. The test using the IEEE 123-bus system is shown and analyzed in Section IV. Section V concludes this paper.

II. Proposed Approach to Non-overlapping Partition

A. Framework

The framework of the proposed approach to the distribution network partition is shown in Fig. 1. It has three steps. First, a new method of electrical distance calculation is given to describe the electric coupling extent in multi-phase networks, which will further work as the weight of edges in the transformed graphs. Secondly, community detection [20], [21] is applied to partition the buses into inner-highly-coupled zones. While it is an over partition process and the results cannot be directly used for the distributed algorithms, it produces significant references for the coupling properties. Thirdly, these little zones are merged into bigger subareas with similar sizes based on the greedy algorithm and tabu search method. Therefore, both the electric connections and size similarities are considered.

B. Electrical Distance

A new real symmetric positive electrical distance is constructed based on branch-current/voltage sensitivity. It can be viewed as an extension of traditional electrical distance definition which is based on Q/V sensitivity [8]. In this paper, the observability for MASE is not considered in the function but will be checked after the partition.

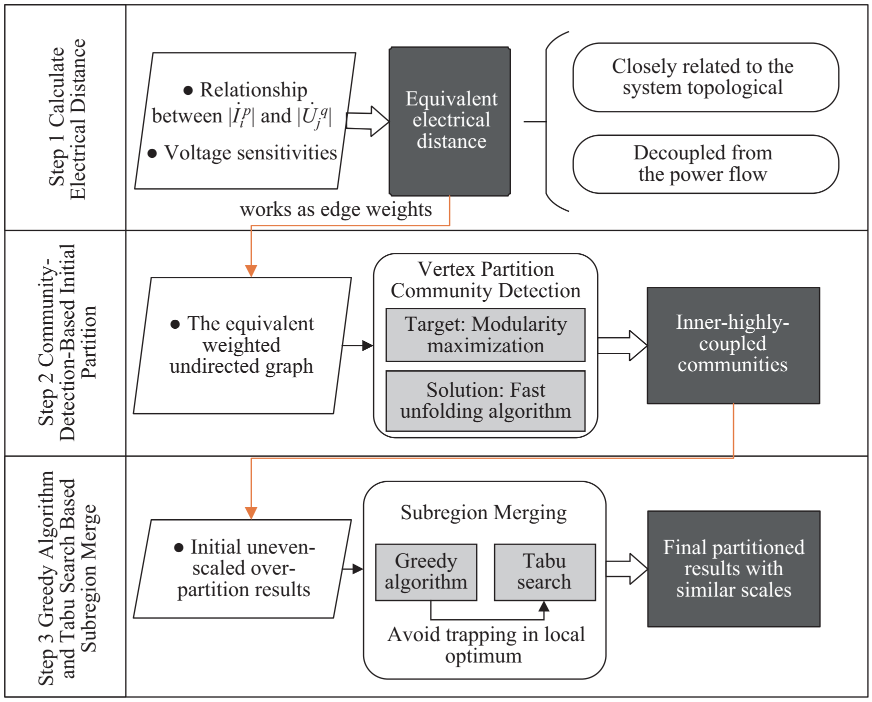

Fig. 1. Framework of non-overlapping partition approach.

The relationship between the current injection phasor at the phase p of bus i, $\dot{\boldsymbol{I}}_i^p$, and the voltage phasor at the phase q of bus j, $\dot{\boldsymbol{U}}_i^p$, can be represented as (1):

$$\dot{\boldsymbol{I}}_i^p = \sum_j \sum_{q=a,b,c} \boldsymbol{Y}_{ij}^{pq} \dot{\boldsymbol{U}}_j^q \tag{1}$$

where, $\boldsymbol{Y}_{ij}^{pq}$ is the element of the admittance matrix related to the phase p of bus i and the phase q of bus j.

Then, for phase p of bus i and phase q of bus j, they are directly connected or line coupled, and their voltage sensitivities can be deduced as (2)–(5):

$$\left|\Delta \dot{\boldsymbol{U}}_i^p\right| = \left|\Delta \dot{\boldsymbol{U}}_j^q\right| \alpha_{ij}^{pq} \tag{2}$$

$$\alpha_{ij}^{pq} = \frac{\left|\Delta \dot{\boldsymbol{U}}_i^p\right|}{\left|\Delta \dot{\boldsymbol{U}}_j^q\right|} = \frac{\left|\partial \dot{\boldsymbol{U}}_i^p\right|}{\left|\partial \dot{\boldsymbol{U}}_j^q\right|} = \frac{\left|\partial \dot{\boldsymbol{U}}_i^p\right|}{\left|\partial \dot{\boldsymbol{I}}_j^q\right|} \Bigg/ \frac{\left|\partial \dot{\boldsymbol{U}}_j^q\right|}{\left|\partial \dot{\boldsymbol{I}}_j^q\right|} \tag{3}$$

$$\left|\Delta \dot{\boldsymbol{U}}_j^q\right| = \alpha_{ji}^{qp} \left|\Delta \dot{\boldsymbol{U}}_i^p\right| \tag{4}$$

$$\alpha_{ji}^{qp} = \frac{\left|\Delta \dot{\boldsymbol{U}}_j^q\right|}{\left|\Delta \dot{\boldsymbol{U}}_i^p\right|} = \frac{\left|\partial \dot{\boldsymbol{U}}_j^q\right|}{\left|\partial \dot{\boldsymbol{U}}_i^p\right|} = \frac{\left|\partial \dot{\boldsymbol{U}}_j^q\right|}{\left|\partial \dot{\boldsymbol{I}}_i^p\right|} \Bigg/ \frac{\left|\partial \dot{\boldsymbol{U}}_i^p\right|}{\left|\partial \dot{\boldsymbol{I}}_i^p\right|} \tag{5}$$

where $|\dot{\boldsymbol{U}}_i^p|$ and $|\dot{\boldsymbol{U}}_j^q|$ are the voltage magnitudes; $|\dot{\boldsymbol{I}}_i^p|$ and $|\dot{\boldsymbol{I}}_j^q|$ are the current injection magnitudes. The above equations indicate that the voltage sensitivities between two buses can be represented as the ratio of current magnitude sensitivity to voltage magnitude sensitivity.

Combined with (1), (3) and (5) can be represented as:

$$\alpha_{ij}^{pq} = \frac{\left|\boldsymbol{Y}_{jj}^{qq}\right|}{\left|\boldsymbol{Y}_{ij}^{pq}\right|}, \alpha_{ji}^{qp} = \frac{\left|\boldsymbol{Y}_{ii}^{pp}\right|}{\left|\boldsymbol{Y}_{ij}^{pq}\right|} \tag{6}$$

Because $\alpha_{ij}^{pq} \neq \alpha_{ji}^{qp}$, in order to obtain symmetrical distances, a new electrical distance definition is raised as:

$$w_{ij}^{pq} = \frac{1}{\alpha_{ij}^{pq}} + \frac{1}{\alpha_{ij}^{qp}} \tag{7}$$

The function w_{ij}^{pq} is positive and symmetric. It is decoupled from the power flow and only depends on the network topology.

Moreover, an extra normalization mathematical step, as shown in (8), is applied in multi-phase distribution networks, for each bus is an undividable integral and all its phases have to be in the same subregion.

$$w_{ij} = \sum_{p,q=a,b,c} w_{ij}^{pq} \tag{8}$$

For node i and node j without direct electrical connections, w_{ij} is set as 0. Thus, the power system model is mapped into an equivalent topological structure $\boldsymbol{G}$ with N vertices, and the topological information is stored in a $N \times N$ adjacency matrix $\boldsymbol{A}$ where $\boldsymbol{A}_{ij} = w_{ij}$.

Therefore, the definition of electrical distance, as proposed in this paper, has the following properties:

1) The electrical distance between two buses with a coupling relationship is always greater than 0.
2) The definition is closely related to the system topological structures.
3) Compared with the currently existing method, the method in this paper is decoupled from the power flows, so it is equally applicable to different power flow scenarios under the same topology and is more feasible.
4) The influence of three-phase imbalance is considered:

a) The sensitivity of all phases is normalized, and the more the number of phases, the stronger the electrical coupling between the two coupled buses.
b) The electrical relationship between the direct-linked phases is far greater than that between the indirect-linked phases.

The value of the electrical distance is primarily determined by that between the same phases in the normalization process, which is consistent with the actual electrical properties.

C. Community-Detection-Based Initial Partition

According to the modularity-based Community Detection algorithm proposed originally in [22] (BGLL algorithm), this paper raises a non-overlapping distribution network partitioning method. By finding highly similar bus groups in the network, the complex network is modularized into high intra-connected and low inter-coupled subregions.

The modularity Q of the weighted undirected network is defined as the following equations:

$$\begin{aligned} Q &= \frac{1}{2m}\sum_{i,j}\left[\boldsymbol{A}_{ij} - \frac{k_i k_j}{2m}\right]\delta(c_i, c_j) \\ &= \frac{1}{2m}\sum_{C\in P}\sum_{i,j\in C}\left[\boldsymbol{A}_{ij} - \frac{k_i k_j}{2m}\right] \end{aligned} \tag{9}$$

$$\begin{aligned} k_i &= \sum_j \boldsymbol{A}_{ij}, m = \frac{1}{2}\sum_i k_i = \frac{1}{2}\sum_{ij}\boldsymbol{A}_{ij}, \delta(u,v) \\ &= \begin{cases} 1, & \text{if } u = v \\ 0, & \text{if } u \neq v \end{cases} \end{aligned} \tag{10}$$

where C represents the community; P is the set of communities in the network; c_i represents the community to which the vertex i belongs. If vertex i and vertex j belong to the same community, then $\delta(c_i, c_j)$ is set as 1, otherwise it is set as 0. k_i is the degree of the vertex i, representing the sum of the weights of all the edges connected with vertex i. m is the sum of the weights of all the edges in the network. The network modularity Q can be regarded as the sum of the modularity of each sub module. The value range of Q is $(-1, 1)$, and the larger the value is, the closer the connection is in the community. Generally, the maximum value of Q is in the range of 0.3–0.7 [16]. $Q = 0$ when the whole network is seen as a single community.

The BGLL algorithm, taking Q in (9) maximization as the optimization objective, is a clustering algorithm that consists of two stages of repeated iteration:

1) Stage 1

For the graph with N vertices, each vertex is first initialized to belong to its independent community, that is, there are N different communities. Then, for each vertex i and each of its adjacent vertices, j, the incremental change of modularity ΔQ is calculated if you move vertex i from its community to that of the vertex j. Afterwards, compare the values and move vertex i to the adjacent community to obtain the largest non-negative ΔQ.

The movement is illustrated in Fig. 2. Move the vertex i from community c_1 to community c_2 where the vertex j is in to form new communities c'_1 and c'_2. If all of ΔQ is less than 0, then vertex i remains in c_1. This process is applied to all vertices in a certain sequence, and the iteration is repeated until there is no more available vertex movement.

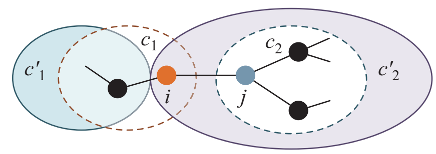

Fig. 2. Framework of non-overlapping partition approach.

The increment of modularity for each vertex movement is represented as:

$$\begin{aligned} Q(c'_1) &= \frac{1}{2m}\sum_{p,q\in c'_1}\left[\boldsymbol{A}_{pq} - \frac{k_p k_q}{2m}\right] = \frac{1}{2m}\cdot \\ &\quad \left[\sum_{p,q\in c_1}\left(\boldsymbol{A}_{pq} - \frac{k_p k_q}{2m}\right) - \left(\boldsymbol{A}_{ii} - \frac{k_i k_i}{2m}\right)\right. \\ &\quad \left. -2\sum_{\substack{q\in c_1 \\ q\neq i}}\left(\boldsymbol{A}_{iq} - \frac{k_i k_q}{2m}\right)\right] \end{aligned} \tag{11}$$

$$\begin{aligned} Q(c'_2) &= \frac{1}{2m}\sum_{p,q\in c'_2}\left[\boldsymbol{A}_{pq} - \frac{k_p k_q}{2m}\right] \\ &= \frac{1}{2m}\left[\sum_{p,q\in c_2}\left(\boldsymbol{A}_{pq} - \frac{k_p k_q}{2m}\right) + \boldsymbol{A}_{ii} - \frac{k_i k_i}{2m}\right. \\ &\quad \left. +2\sum_{q\in c_2}\left(\boldsymbol{A}_{iq} - \frac{k_i k_q}{2m}\right)\right] \end{aligned} \tag{12}$$

$$\begin{aligned} \Delta Q &= Q(c'_1) + Q(c'_2) - Q(c_1) - Q(c_2) \\ &= \frac{1}{m}\sum_{q\in c_2}\left(\boldsymbol{A}_{iq} - \frac{k_i k_q}{2m}\right) - \frac{1}{m}\sum_{q\in c_1}\left(\boldsymbol{A}_{iq} - \frac{k_i k_q}{2m}\right) \\ &\quad [1 - \delta(i,q)] \end{aligned} \tag{13}$$

2) Stage 2

Each community in the result of Stage 1 is regarded as a vertex. Therefore, a new graph is formed. The weight of the edge between the new vertices is set to equal the sum of the weight of the edge between the previous two communities. The weight of the self-loop of the new vertices is twice the weight of all the edges in the previous community. The sum of degrees in the whole graph remains unchanged. Then the process in Stage 1 is employed again until the communities do not change.

The system is divided into multiple highly cohesive non-overlapping subregions, identified as *community*. Although it reflects the strength of the coupling extent in the distribution network, such an algorithm has a common drawback: it is an over-partition process and the partitioned subregions are too small and scaled unevenly. Thus, a merge process needs to be further applied on the initial partition result.

D. Subregion Merging

As mentioned in the Introduction section, it is a key premise to ensure the computational efficiency of distributed algorithms. Therefore, it is necessary to merge the initially partitioned subregions into a reasonable number according to this principle. In this paper, the number is preset.

It can be seen as a k-balanced graph partition problem [23], [24]. It refers to dividing a weighted graph into non-overlapping k parts with as few similar scales as possible, which is the constraint, to obtain the minimized sum of edge weights connecting different subregions, namely the cut set, which is the optimization objective. When the edges are unweighted, the problem is equivalent to taking the size similarity of subregions as the optimization objective.

In this paper, the subregion merging problem is constructed as a k-balanced graph partition problem. It is solved in three-phases: equivalent graph construction phase, a coarsening phase by a greedy algorithm, and a refining phase by a tabu search algorithm.

1) Equivalent Graph Construction

First, an equivalent graph for a k-balanced graph partition is constructed. After the initial partition, each *community* is regarded as a weighted vertex and the weight is set as the scale of "community" such as the number of nodes or phases, etc. The edges connecting the "communities" are viewed as non-weighted, because the merging part focuses more on the scale-similarity. The size of the equivalent graph is much smaller than the original. This paper primarily considers the radial distribution network, so it is sparse and has limited edges.

For the distributed algorithms in power networks, the interaction information between subregions is limited, for example, the information of only interacting boundary nodes [9], [25], therefore, the communication cost is not considered in this paper.

In this way, the problem of subregion merging is transformed into the problem of k-balanced graph partition with weighted points and unweighted edges.

2) Coarsening Phase

A greedy algorithm based coarsening phase is used here to generate a good initial solution as the input of the following tabu-search-based refining phase.

In this k-balanced graph partition problem of a weighted-vertices unweighted-edges graph, the optimization problem can be viewed as the minimization of *scale imbalance*, the rate of the subregions' biggest size and minimum size, in (14).

$$\min \quad f = \frac{\max(N_i)}{\min(N_i)},\ i = 1, 2, \cdots, k \tag{14}$$

where k refers to the preset number of subregions, N_i is the size of the ith subregion.

In a non-overlapping partition, the weight of each vertex and the final partitioned number is known in advance. So, the optimization problem equals (15).

$$\min \quad f = \sum |N_i - \overline{N}|,\ i = 1, 2, \cdots, k \tag{15}$$

$$\overline{N} = \frac{1}{k} \sum w \tag{16}$$

where, $\overline{N}$ is the average size of the subregions.

A greedy algorithm is applied to distribute the overall objective into a smaller one:

Algorithm 1: Greedy Algorithm for Coarsening Phase

Input: $G := \{V, E\}, \{K\}, \{W\}, k_{\min} := \min\{K\}$;
1 Initialize the counter of subregions $kc := 0$;
2 Initialize the candidate vertices set $\{S\} := \{v | k(v) = k_{\min}, v \in \{V\}\}$
3 **for** *each* $v_i \in \{S\}$ **do**
4   **if** v_i *can be cut as a subregion* **then**
5   Do the cut and update $kc, \{V\}, \{K\}, \{W\}\{S\}, k_{\min}$
6   **if** $k_{\min}$ *changes* **then**
7   Return to step 3;
8   **end**
9   **end**
10 **end**
11 Initialize the related counter set of $\{S\} : \{SF\} := \{f(v) | v \in \{S\}\} = \{0\}$
12 **while** $\{S\}$ *is not empty* **do**
13   Find the $v_m \in \{V\} : k(v_m) = k_{\min}$ and $f(v_m) = \min\{SF\}$
14   **if** v_m *cannot be merged into its neighour*(v') *and* $f(v_m) = 0$ **then**
15     $f(v_m) := 1$ and update $\{SF\}$;
16   **else**
17   Do the merge and update $\{S\}, \{SF\}, \{W\}, \{K\}$
18   **if** $k(v') < k_{\min}$ **then**
19   Go to step 3
20   **else**
21       $k(v') = k_{\min}$
22       $v_m := v'$, go to step 10;
23   **end**
24   **end**
25 **end**

Starting from vertex v with the minimum degree and minimum weight, search the connected vertex set S_v to make the weight sum of vertex v and v' most closer to the objective value. That is, whether there is a unique solution to obtain a minimum negative value in (17).

$$\begin{aligned} \min \quad & \Delta(v_n) = |w(v) + w(v_n) - \overline{N}| - |w(v) - \overline{N}|, v_n \in S_v \\ \text{s.t.} \quad & \begin{cases} \min \Delta < 0 \\ |v'|_{\Delta(v') = \min \Delta} = 1 \end{cases} \end{aligned} \tag{17}$$

If such vertex v' exists and is unique, then merge vertex v into v' and make the sum of their weights to be that of the new vertex.

If it exists but is not unique, then leave the graph unchanged and start from the vertex with the minimum degree and the minimum weight except v.

If there is not a negative Δ, which means that merging v into any vertex in S_v will not make its weight closer to the target value $\overline{N}$, then cut vertex v off as a subregion.

These steps should iterate until you obtain the preset number of subregions.

3) Refining Phase

In order to escape the trap of local optimum of the greedy algorithm and to achieve the global partition equilibrium, a heuristic algorithm, tabu search, is used to refine the results.

The solution of the coarsening phase is taken as the initial current optimum and the initial historical optimum. It is intensified and diversified by re-decomposing and re-merging the vertices with two move operators repeatedly. Equation (14) works as the evaluation function for measuring the attractiveness of the results.

Specifically, the two move operators respectively are the neighborhood move operator and the exchange move operator. *Move gain* is introduced here to describe the partition improvement of each movement. one of its neighboring subregions from its previous one. Start from any one subregion S_m except for the maximum weight subregion $S_{\max}$ that $S_m \in \{S_i | S_i \neq S_{\max} S_i \subset \{S\}\}$ and randomly select a border node v_n connecting to S_m from its adjacent subregion $S_n (m \neq n)$. Move v_n into S_m from S_n so that the vertices left in S_n are interlinked. Its move gain is described in (18).

$$\Delta f_1 = |N_m + w_{v_n} - \overline{N}| + |N_n - w_{v_n} - \overline{N}| - |N_m - \overline{N}| - |N_n - \overline{N}| \tag{18}$$

Exchange move operator references that after moving the above mentioned neighboring move operator v_n into S_m from S_n, then randomly select a border node v_m in S_m adjacent to S_n and move it into the S_n so that the left subregion S_m remains interlinked. Its move gain is described in (19).

$$\Delta f_2 = |N_m + w_{v_n} - w_{v_m} - \overline{N}| - |N_m - \overline{N}| + |N_n + w_{v_m} - w_{v_n} - \overline{N}| - |N_n - \overline{N}| \tag{19}$$

Consider all possible movements and execute that one with the minimum move gain. If v_n is moved into S_m from S_n, then v_n is rejected from being moved back into S_n in the given number of following steps, namely the *tabu length*, unless it satisfies the *amnesty criterion*. For example, in this paper, set the tabu length as tt, then the size of the tabu list is $2tt$. Each time there is a movement, the corresponding data $[v_n, S_n]$ is added to the end of the list and the first row of the list is deleted. If the element $[v_n, S_n]$ exists in the list, then the movement of v_n into S_n is rejected.

Amnesty criterion references that when the current optimal solution is better than the historical optimal solution but is on the tabu list, it should be released and ignored and be accepted directly.

The neighborhood exploration strategy is described as the following. Make a candidate set of all possible move operators, calculate their corresponding move gain, and arrange them in order. The current best move operator is called the current optimal solution. If the current optimal solution is not rejected, or is rejected but meets the amnesty criterion, then execute it. Otherwise, consider the suboptimal solution until a feasible solution is found. If it is better than the historical optimal solution changes, update it, including the corresponding evaluation value and the detailed partition scheme. If the current optimum is the same as the historical optimum, but the partition schemes are different, it means that it has multi-solutions, so add the new solution to the historical optimal scheme record.

There are three types of termination criterion defined in this paper. The first one is to set the maximum search times $t_{\max}$. If the counter number exceeds $t_{\max}$, the current optimal solution will be used as the final result. The second one is to specify the number of steps t_{stop}. If the current optimal solution does not change within t_{stop} steps, then the search is terminated. The third one is that when currently there are no available move operators, then the search is terminated. The result is the global optimal partition scheme with the minimum evaluation function value.

III. Proposed Approach to Border-Bus Overlapping PartitioN

The tie-line overlapping partition can be directly obtained by extending the non-overlapping partition pattern as presented in [10]. Another important partition pattern in a distribution network, which many MASE methods are based on, is the border-bus overlapping partition, which can be obtained by the following method.

A. Framework

The framework of the proposed approach for the border-bus overlapping partition is similar to that of non-overlapping Partition Approach, however, you just replace the vertex partition community detection with the link partition community detection.

Combining the community discovery method based on edge division [26], [27], and the merging method based on the principle of uniformity of subregions, this paper proposes a node overlapping distribution network partitioning method.

B. Community-Detection-Based Initial Partition

According to the links partitions in [28], the vertices and the links are equivalently conversed and then the modularity-based community detection is applied in this initial partition stage.

For a graph of a distribution network with N vertices and L links, by shifting vertices to links and links to vertices, an adjacency matrix of the line graph, $\boldsymbol{C}$, is defined. First, construct the correlation matrix $\boldsymbol{B}$:

$$\boldsymbol{B}_{i\alpha} = \boldsymbol{B}_{j\alpha} = w_{ij} \tag{20}$$

where α refers to the edge with the ends i and j. Matrix $\boldsymbol{B}$ can be regarded as the adjacency matrix of this bilateral network, which contains all the graph's information. For example, the degree of node i, k_i, and the number of vertices connecting to the line α, k_α, can be expressed as:

$$k_i = \sum_{\alpha} \boldsymbol{B}_{i\alpha} k_\alpha = \sum_{i} \boldsymbol{B}_{i\alpha} \tag{21}$$

Then the adjacent matrix $\boldsymbol{C}(L \times L)$ is defined as:

$$\boldsymbol{C}_{\alpha\beta} = \sum_{\substack{i, k_i > 0 \\ j, k_j > 0}} \frac{\boldsymbol{B}_{i\alpha} w_{ij} \boldsymbol{B}_{j\beta}}{k_i k_j} \tag{22}$$

$\boldsymbol{C}$ can be regarded as a weighted undirected network with L vertices as well as self-loops. For a large and sparse distribution network, the number of branches is limited, so the construction of the adjacency matrix $\boldsymbol{C}$ will not cause a massive increase in the dimension of the matrix.

Therefore, matrix $\boldsymbol{C}$ can be directly put into (13) for the initial partition. This link partitioning method ensures that there is only one layer of border nodes in the overlapping areas. However, it is still an over-partition process.

C. Subregion Merging

Similarly, in this part, the subregion merge problem is constructed as a k-balanced graph partition problem with the optimization goal of scale similarity. It is solved by a greedy algorithm and a tabu search method.

1) Equivalent Graph Construction

First, construct an equivalent graph with weighted vertices and unweighted edges. Each subregion of the result in the previous initial partition is viewed as a weighted vertex, and the weight is set as its size. Considering the three-phase imbalance of distribution networks, here we take the number of phases as the size. The adjacent subregions are connected by edges. Additionally, the overlapping areas are marked as a different type of weighted vertices in the graph and are connected with the subregions to which they belong with a different type of edges.

2) Coarsening Phase

A greedy algorithm is used to pursue the scale uniformity of a target number of subregions. Different from the non-overlapping partitioning method, in the border-node overlapping pattern, the existence of overlapping areas will cause the uncertainty of the average size of optimally partitioned subregions. But the influence is limited. So, in this section, the average size $\tilde{N}$ is approximated to $\overline{N}$.

Since it involves the repeated calculation of the size in the overlapping area and the locally connected graph, the sub objective of the greedy algorithm can be described as:

$$\begin{aligned} \min \quad & \Delta(v_n) = |w(v) + w(v_n) - w(v \cap v_n) - \tilde{N}| \\ & \qquad - |w(v) - \tilde{N}|, v_n \in S_v \\ \text{s.t.} \; & \begin{cases} \min \Delta < 0 \\ |v'|_{\Delta(v')=\min \Delta} = 1 \end{cases} \end{aligned} \tag{23}$$

where $w(S_m \cap v_n)$ refers to the weight of the overlapping part of the vertex v_n and the subregion S_m.

Starting from vertex v with the minimum degree and the minimum weight, search the connected vertex set S_v to determine whether there is a vertex v' in S_v to make the weight of the new subregion more closer to the objective value. That is, whether there is a unique solution to obtain a minimum negative value in (23).

If such vertex, v', exists and is unique, then merge vertex v into v' and update the new weights.

If the solution exists but is not unique, then keep the graph unchanged and restart the search from the vertex with the minimum degree and the minimum weight, except v.

If there is not a negative Δ, which means that merging v into any vertex in S_v will not make its weight closer to the target value $\tilde{N}$, then cut the vertex v off as a subregion.

3) Refining Phase

A tabu search is used to refine the partition results. Define the scale imbalance of the neighboring subregion i and j as (24).

$$g_{ij} = \max\{N_i/N_j, N_j/N_i\} \tag{24}$$

Equation (25) is the evaluation function. The optimization goal is to minimize the maximum value of the scale imbalance between all adjacent subregions. The merging result in the coarsening phase is taken as the initial solution as well as the current optimal solution, $f_{\text{cur}} = \underline{f}_{\text{best}}$.

$$\min \quad f = \max(g_{ij}) \tag{25}$$

There are two move operators, the neighborhood move operator and the exchange move operator, respectively. Their move gains are given as the following.

For the neighborhood move operators, move v_n into S_m from S_n, and its move gain is shown in (28).

$$a = N_m + w_{v_n} - w(S_m \cap v_n) \tag{26}$$

$$b = N_n - w_{v_n} + w((S_n - v_n) \cap v_n) \tag{27}$$

$$\Delta g_1 = \max\{a/b, b/a\} - \max\{N_m/N_n, N_n/N_m\} \tag{28}$$

where $(S_n - v_n)$ refers the subregion S_n after removing v_n out.

For the exchange move operators, move v_n into S_m from S_n and move v_m into S_n from $S_m (v_m \neq v_n)$ so that S_m and S_n are still connected internally. The move gain is shown in (31).

$$\begin{aligned} a = & N_m - w_{v_m} + w((S_m - v_m) \cap v_m) + w_{v_n} \\ & - w(S_m \cap v_n) \end{aligned} \tag{29}$$

$$\begin{aligned} b = & N_n - w_{v_n} + w((S_n - v_n) \cap v_n) + w_{v_m} \\ & - w(S_n \cap v_m) \end{aligned} \tag{30}$$

$$\Delta g_2 = \max\{a/b, b/a\} - \max\{N_m/N_n, N_n/N_m\} \tag{31}$$

IV. Case Study

The proposed partitioning method is developed in the MATLAB R2018b environment. This section gives the results and comparisons of numerical simulations that have been performed on a computer with an Intel (R) Core (TM) i5-8400 CPU @ 2.80 GHz and 8 GB main memory.

A. Equivalent Graph Construction

In this paper, when constructing the equivalent topology of the distribution network, the buses at both ends of the closed circuit breakers are simplified to one vertex. The isolated part of the disconnected network is no longer included in the equivalent topology to be partitioned.

Taking the IEEE 123-bus distribution system [29] as an example, the electrical system model is shown in Fig. 3. Its renumbered equivalent weighted undirected graph is shown in Fig. 4.

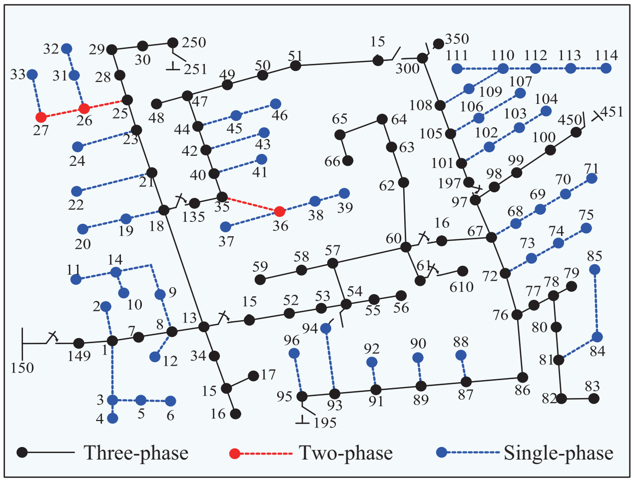

Fig. 3. IEEE 123-bus power system mode.

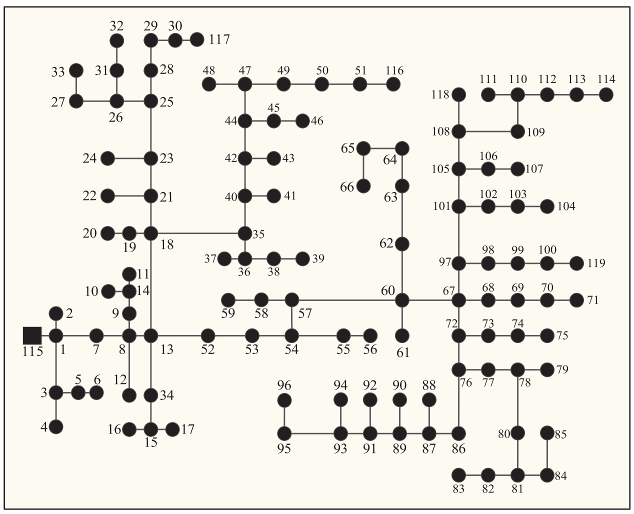

Fig. 4. IEEE 123-bus equivalent weighted undirected topology.

Taking the three-phase lines between vertex No. 1 and No. 7, the related elements in the admittance matrix is shown in (32). The calculated electrical distance is shown in (33).

$$\begin{bmatrix} 9.8803-20.4871i & -4.7993+7.0327i & -1.7318+4.5682i \\ -4.7993+7.0327i & 11.0484-21.2300i & -2.9397+5.5754i \\ -1.7318+4.5682i & -2.9397+5.5754i & 8.8766-19.7719i \end{bmatrix} \tag{32}$$

$$\begin{bmatrix} 0.9714 & 0.3562 & 0.2129 \\ 0.2923 & 0.8009 & 0.2219 \\ 0.1832 & 0.2309 & 0.8318 \end{bmatrix} \tag{33}$$

The electrical distance between nodes is 4.1016, the sum of all elements in (33), in which the diagonal elements account for $2.3041/4.1016 = 63.5\%$ and the non-diagonal elements account for $1.4975/4.1016 = 36.5\%$. Therefore, the electrical distance between the same phases is dominant, proving that the proposed method is consistent with the actual electrical properties.

B. Non-Overlapping Partition

1) Initial Partition Result

The initial non-overlapping partition result of the IEEE 123-bus network based on community detection is given in Fig. 5. It is partitioned into 15 small communities with uneven scales. The number of nodes of each subregion varies from 2 to 18 and that of the phases varies from 4 to 38. Therefore, it cannot be used as the final result in distribution networks.

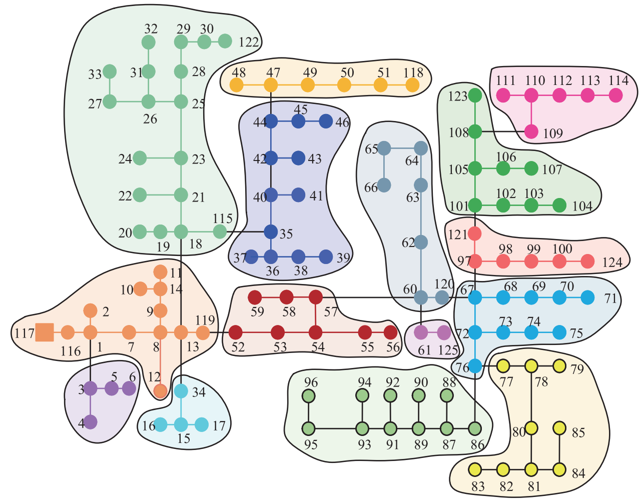

Fig. 5. IEEE 123-bus non-overlapping initial partition result based on community detection.

2) Equivalent Graph Constructed for Subregion Merging

Construct Fig. 5 into an equivalent graph with weighted vertices and unweighted edges for applying the following subregion merging algorithms. It is shown in Fig. 6 and the numbers in the circles are the weight.

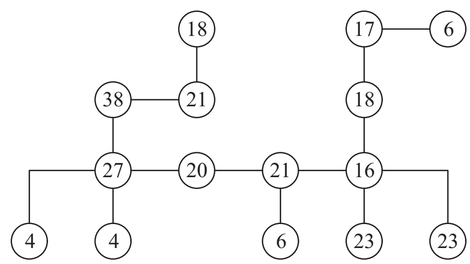

Fig. 6. IEEE 123-bus non-overlapping equivalent graph constructed for subregion merging.

Taking the target number of subregions $N = 6$ and $N = 4$ respectively as examples, the IEEE 123-bus system is partitioned into non-overlapping subregions. The tabu length is set as $tt = 3$, the maximum search times is $t_{\max} = 200$, and the specified number of steps is $t_{\text{stop}} = 3$. The results are presented in Figs. 7 and 8.

The proposed method is computationally efficient. When merging the network into 6 subregions, the scenario under consideration (the number of merge/split judgments) is 23 in the coarsening phase and the ones considered in the refining phase is 5. Compared with $C_{14}^{6} = 3003$ times in the traversing method, it is $(3003 - 28)/3003 = 99.1\%$ less. When merged into 4 subregions, the scenario considered in the proposed method is $22 + 2 = 24$ while in the traversing method the number is $C_{14}^{6} = 3003$ times. It is $(364 - 24)/394 = 93.4\%$ less.

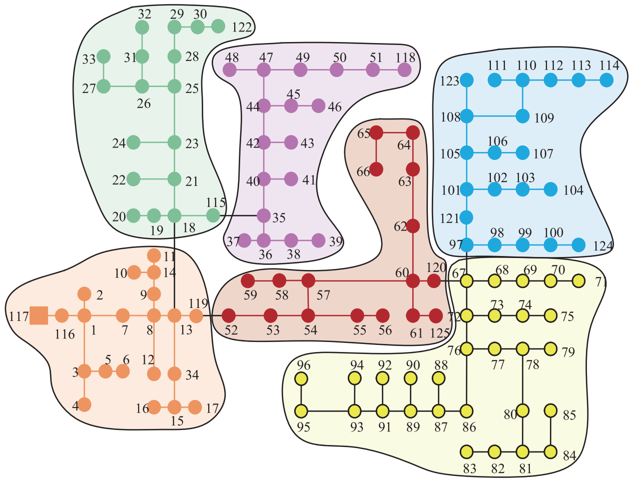

Fig. 7. IEEE 123-bus non-overlapping partition ($N = 6$).

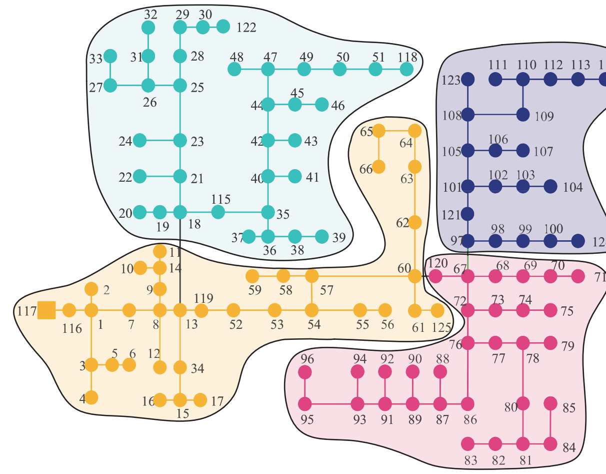

Fig. 9. IEEE 123-bus over-overlapping partition result in [13].

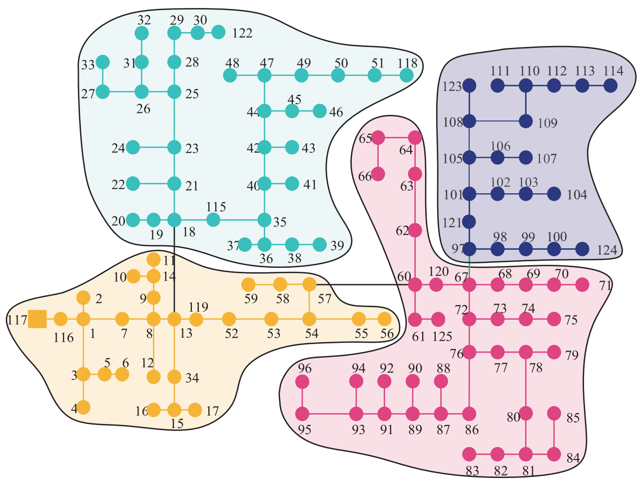

Fig. 8. IEEE 123-bus non-overlapping partition ($N = 4$).

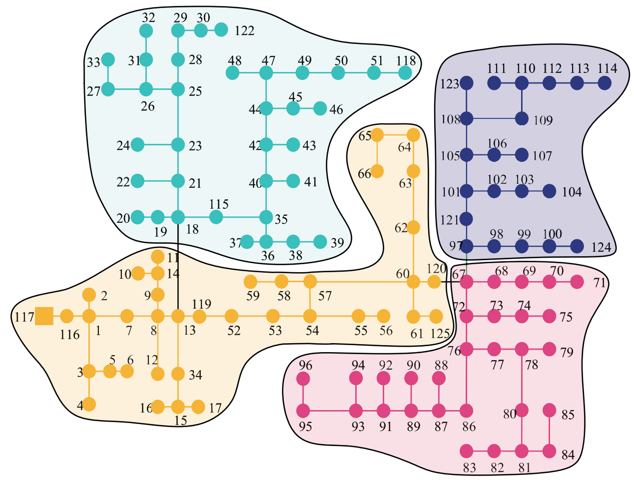

Fig. 10. IEEE 123-bus over-overlapping partition result in [28].

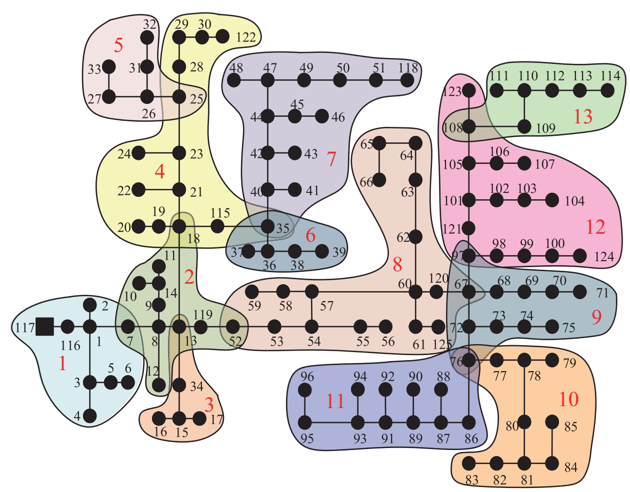

Fig. 11. IEEE 123-bus over-overlapping initial partition result based on community detection.

The result in the refining phase of the Subregion Merging step is the same as that of the traversing method, which verifies the global optimality of the proposed tabu-search-based method. Comparing the result of 4 subregions with the existing literature, it is the same as that in [11] and [17], and is similar to Fig. 9 in [15] and Fig. 10 in [30]. Reference [11] partitions the network based on node number similarity and observability for MASE, but neglects the electrical coupling relationship. The methods in [15] and [17] are both based on the assumption of three-phase balance and is sensitive to the power flow selected for the calculation. While in this paper, the three-phase imbalance is considered, and the result is decoupled from the power flows. Reference [30] divides the training data into smaller packages for MASE, and just mentions the nodes scale similarity without more descriptions, while this paper provides theoretical explanations and detailed algorithms. Therefore, this proposed method is effective and credible.

C. Border-Bus Overlapping Partition

1) Initial Partition Result

The initial border-bus overlapping partition result of the IEEE 123- bus network is shown in Fig. 11. It has the apparent shortages of over-partition and uneven scales.

2) Equivalent Graph Constructed for Subregion Merging

Figure 11 is constructed into an equivalent graph with

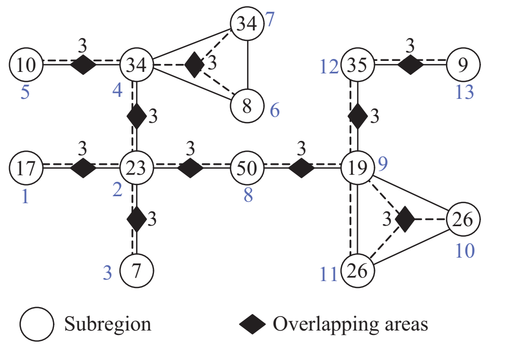

Fig. 12. IEEE 123-bus border-node overlapping equivalent graph constructed for subregion merging.

weighted vertices and unweighted edges as shown in Fig. 12 for applying the following subregion merging algorithms. The circles are the equivalent vertices of the subregions and the circled number is the weight. The blue numbers beside the circles are the serial numbers of the vertex. The diamonds are the equivalent vertices of the overlapping areas and the numbers nearby are the weights. The solid black lines connect adjacent subregions, and the dotted lines connect the overlapping areas and the subregions to which they belong.

3) Final Partition Result

Taking the target number of subregions $N = 6$ and $N = 4$ respectively as examples, partition the IEEE 123-bus system into border-bus overlapping subregions. Set $tt = 3$, $t_{\max} = 200$ and $t_{\text{stop}} = 3$. The results are presented in Figs. 13 and 14.

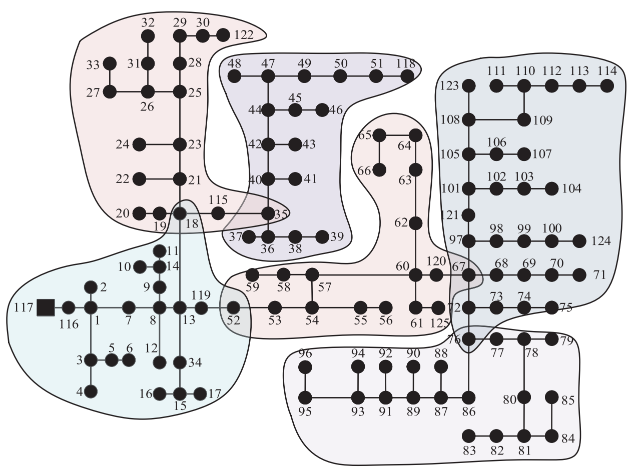

Fig. 13. IEEE 123-bus border-node overlapping partition ($N = 6$).

The final partition result is the same as that by the traversing method. When merged into 6 subareas, the proposed method considers $23 + 3 = 26$ scenarios in total while the traversing method considers 3705 times. When merged into 4 subareas, the scenarios considered are $23{+}2 = 25$ and 1710 respectively.

Comparing the partition results of 4 subregions in Fig. 14 with that in [12], [30], [31] as shown in Fig. 15, they are similar. Thus, the credibility of the proposed method is verified. Different from the simple subjective partitioning method in [31], this paper provides strong algorithm support. It takes both the electrical coupling relationship and the scale similarity into account, which is realized by the method in Section III. While the considered elements are different from the data-package-division method in [30] and the node number similarity and observability based method in [12], the results verify that this proposed method is effective and credible.

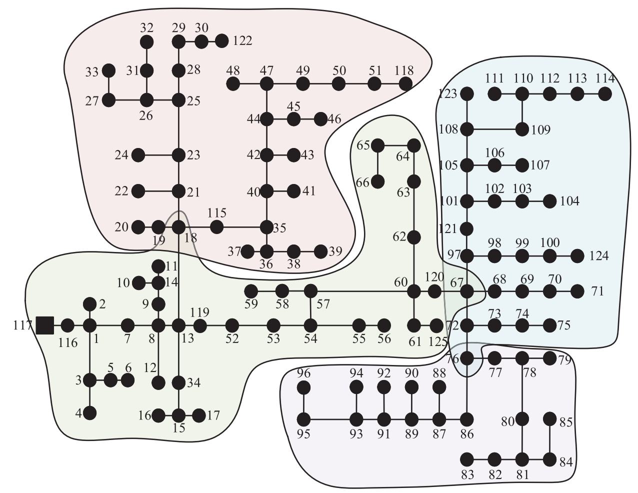

Fig. 14. IEEE 123-bus border-node overlapping partition ($N = 4$).

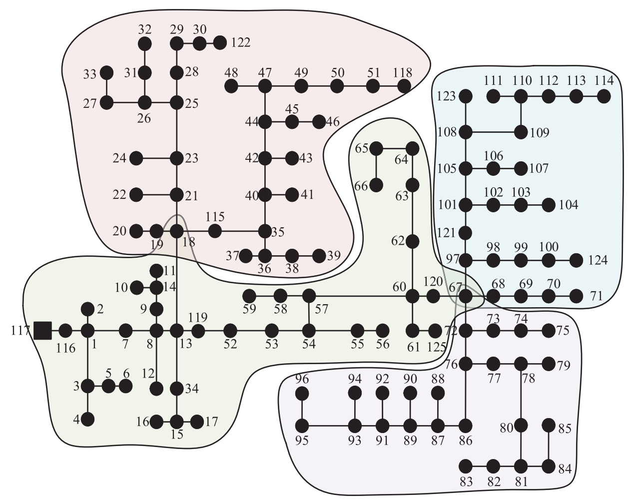

Fig. 15. IEEE 123-bus border-node overlapping partition ($N = 4$) in other literatures.

4) Partition Efficiency

To prove that the proposed method is efficient compared with other methods, Monte Carlo simulations are used for the test of this part, 100 trails for each test, to compare the efficiency of the method proposed in [12], the traversing method and the method in this paper.

These three methods are used to partition the IEEE123-bus system into four border-node overlapping subregions. As shown in Fig. 16, the red curve represents the time-consuming of the traversing method, the blue curve represents the time-consuming of the method in [12], and the green curve represents the time-consuming of the method in this paper. Through 100 partition experiments, it can be found that the method proposed in this paper can greatly shorten the time spent in the partition of the distribution system. The results show that the proposed method is efficient.

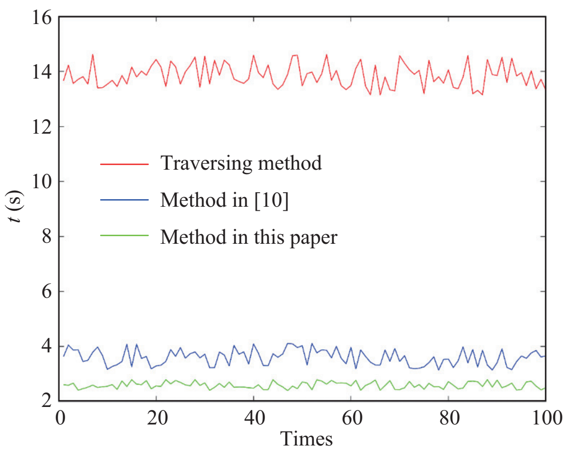

Fig. 16. Time consuming of three partitioning methods.

V. Conclusion

This paper has proposed a community-detection-based non-overlapping and border-node overlapping partition approach in three-phase imbalanced distribution networks. First, a novel electrical distance is defined to quantify the coupling relationship of buses and then is used as the edge weight of the equivalent topology graph. Afterwards, the vertex-partition and link-partition community detection algorithms are introduced in the initial over-partition stage to generate high-inter-coupled subregions. Finally, a greedy algorithm and a tabu search method are employed to merge these small subregions to the target number according to the scale similarity. The results and comparisons of the tests on IEEE 123-bus systems verify that the proposed partition approach is effective and credible.

This approach is equivalently applicable for different scenarios in distribution networks, such as MASE and distributed control. It is based on the coupling relationship of buses and the scale similarity with strong theoretical and algorithm support. Three-phase imbalance is fully considered. The partition result is decoupled with the power flow and only relies on the network parameter.

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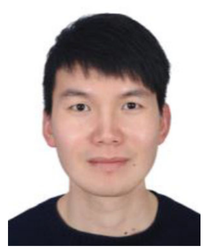

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